

## A UNIFYING METHODOLOGY FOR ROBUST HIGH ORDER CFD: SUMMATION-BY-PARTS

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### ABSTRACT

Summation-by-parts (SBP) is originally a concept from the finite difference community with the goal to mimic finite element energy analysis techniques. In recent years, this simple framework has conceptually exploded enabling a unifying framework for the stability analysis of finite difference (FD), finite volume, and discontinuous Galerkin (DG) methods on structured and unstructured polytope meshes for linear and non-linear conservation laws on conforming and non-conforming grids. The most important consequence of SBP is that it naturally guides the path to stability and robustness as it mimics continuous stability analysis. The strong theoretical foundation of the SBP methodology allows the construction of flexible high-order numerical approximations that are robust for complex multi-scale applications, e.g. compressible turbulence. This robustness is guaranteed via discrete entropy stability. However, the SBP framework can incorporate other desirable properties that are potentially important for CFD: For instance, it is possible to construct split-form SBP methods, for the compressible Euler equations, that are fully conservative and kinetic energy preserving. Remarkably, these methods are virtually dissipation free but robust for e.g. underresolved turbulence. These new developments offer exciting novel ways of incorporating sophisticated turbulence modelling in e.g. the DG framework. Moreover, these robustness investigations guide the way to the construction of provably stable complex boundary conditions for fluid dynamics.

This mini symposium will bring together experts from different disciplines that push the boundaries of the SBP related methods for CFD to new frontiers with talks on adaptivity, HPC, non-linear robustness, high order turbulence modeling, boundary conditions, efficient time integration for high-order methods, meshing technologies, and general new developments of the SBP concept itself.