

## MONOTONICITY PRESERVING FINITE ELEMENT METHODS FOR TRANSPORT EQUATIONS AND HYPERBOLIC SYSTEMS

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### ABSTRACT

The main focus of this minisymposium is on new approaches to enforcing monotonicity preservation in finite element (FE) methods for conservation laws. Conventional stabilization techniques deliver optimal rates of convergence in smooth regions but may fail to suppress spurious oscillations in the vicinity of steep fronts. The last five years have witnessed an increased interest of the finite element CFD community in shock-capturing techniques based on discrete maximum principles and their generalizations to hyperbolic systems. Significant advances have been made in the development and theoretical justification of limiting techniques for continuous and discontinuous Galerkin approximations on unstructured meshes. The mathematical theory of algebraic flux correction methods was enriched by rigorous proofs of monotonicity, existence, and uniqueness of solutions to nonlinear discrete problems. Proofs of linearity preservation and Lipschitz continuity were obtained for existing and new limiter functions. A priori error estimates were derived for singularly perturbed convection-diffusion equations. The efficiency of iterative solvers was improved using differentiable limiters and convergence acceleration techniques. The concept of invariant domains was introduced as a criterion of monotonicity preservation for nonlinear conservation laws and hyperbolic systems. Moreover, flux-corrected transport (FCT) algorithms for arbitrary high order finite elements were developed using Bernstein polynomials as local basis functions. This minisymposium will explore further theoretical and numerical developments for the above methods and consider applications of these techniques to transport equations and hyperbolic systems.