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A FINITE ELEMENT APPROACH FOR HYDROELASTIC VIBRATIONS OF PARTIALLY FILLED PRESTRESSED ELASTIC TANKS

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Abstract. A Finite Element approach is presented for the computation of linearized hydroelastic vibrations of partially filled tanks, around a prestressed state. The focus is given on the competition between the prestressed and added-mass effects on the dynamic behavior of the fluid-structure system. First, by taking into account geometric non-linearities, a quasi-static solution of a tank loaded by hydrostatic follower forces is computed [1]. Then, the linearized hydrostatic vibrations around the prestressed configuration are computed with a standard fluid-structure formulation [2, 3]. The methodology is finally applied on a cylinder partially filled with liquid, showing very good agreements between our numerical results and experiments from [4].

1 INTRODUCTION

This study deals with the Finite Element (FE) computation of hydroelastic vibrations of prestressed elastic tanks with free-surface fluid. The prediction of fluid-structure dynamic behavior is a critical step in aerospace engineering for the design of launchers with liquid propellant or tanks of satellites [2, 3]. The use of flexible structures, such as hyperelastic membranes or very thin walls, induces the need of numerical models taking into account the prestressed state due to geometrical non-linearities. The main objective of this work is to estimate the influence of the prestressed state on the dynamic behavior of the fluid-structure system. The proposed approach consists (i) in solving the quasi-static non-linear FE problem of the filled tank submitted to hydrostatic follower forces [1], and then (ii) to evaluate the hydroelastic vibrations around the prestressed state. For the dynamical problem, eigenmodes are evaluated around the prestressed state taking into account the added mass effect of the incompressible fluid (Figure 1). Some numerical examples are proposed (i) to validate the model by comparison with some experimental results from the literature [4] and (ii) to show the efficiency of the approach. The competition between the added-mass and prestressed effects of a partially filled tank with elastic bottom, for various fluid heights, is highlighted through parametric studies.

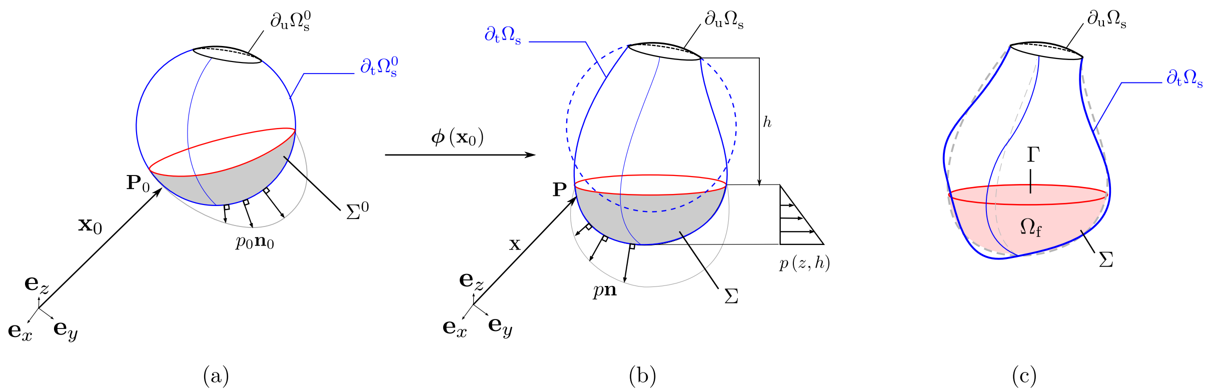

Figure 1: (a) Reference configuration of the internal surface of a tank ; (b) Current configuration of internal surface of a tank loaded by hydrostatic follower forces; (c) Linearized hydroelastic vibration around a pre-stressed state without gravity effects

1.1 Hypotheses

Solid hypotheses The structure is supposed homogeneous isotropic elastic and pre-stressed by hydrostatic follower forces. For the geometrical nonlinear model, we consider the Saint-Venant Kirchhoff constitutive law which is valid for large displacement and small strains. The total structural displacement $\mathbf{u}_t$ is the sum of two terms: a quasi-static non linear solution $\mathbf{u}_s$ and the displacement fluctuation around the prestressed state $\mathbf{u}$, such that

$$\mathbf{u}_t = \mathbf{u}_s + \mathbf{u} \tag{1}$$

Note that the displacement fluctuation amplitude is supposed very small compared to a characteristic length of the structure (e.g. the thickness of the tank).

Fluid hypotheses The fluid is supposed heavy, inviscid, irrotational and incompressible. Moreover, fluid surface tension is neglected which implies that the free surface is plane at the equilibrium. The sloshing vibrations due to gravity effects are not considered in the study. Consequently, the pressure fluctuation p on the free surface Γ is given by

$$p = 0 \quad \text{on} \quad \Gamma \tag{2}$$

With all the previous hypotheses, the pressure fluctuation around the prestressed state can be either defined as the eulerian or by the lagrangian fluctuation (see [2] for details)

$$p = p_L = p_E \tag{3}$$

Fluid-structure interface hypotheses The no-penetration condition at the fluid-structure interface implies that the fluid normal velocity $\mathbf{v}$ is equal to the structural normal velocity $\dot{\mathbf{u}}$:

$$\mathbf{v} \cdot \mathbf{n}_\mathrm{f} = -\dot{\mathbf{u}} \cdot \mathbf{n}_\mathrm{s} \quad \text{on} \quad \Sigma \tag{4}$$

where Σ is the fluid-structure interface (also called the wetted surface) and $\mathbf{n}_\mathrm{f}$ (resp. $\mathbf{n}_\mathrm{s}$) is the outer unitary normal to the fluid domain (resp. to the solid domain).

2 HYDROELASTICITY WITHOUT PRESTRESSING

2.1 Coupled formulation

A standard $(\mathbf{u}, p)$ formulation [2] is chosen where $\mathbf{u}$ is the displacement fluctuations around a reference configuration and p the pressure fluctuations in the fluid domain (Figure 2).

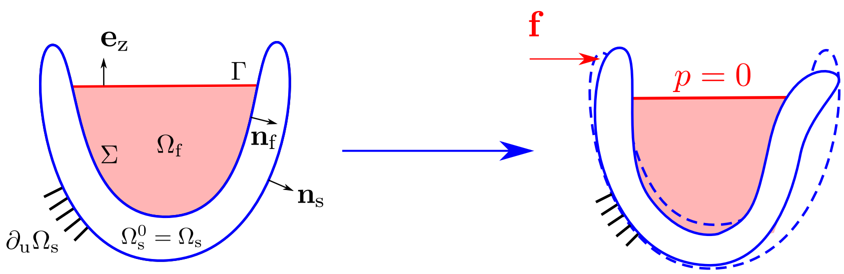

Figure 2: Hydroleastic problem description without pre-stress effect under a small amplitude harmonic load $\mathbf{f}$, with no gravity effect such that the pressure fluctuation is fixed as $p = 0$ on the free surface Γ.

A volumetric discretization is generated for each domain (Figure 3) with a coincident mesh at the fluid-structure interface Σ^h. Let's define $\mathbf{u}^\mathrm{h}$ and p^h as a linear combination of shape functions on the discretized domains written as

$$\mathbf{u}^\mathrm{h} = \mathbf{N}_\mathrm{q}\mathbf{q} \quad \text{and} \quad p^\mathrm{h} = \mathbf{N}_\mathrm{p}\mathbf{p} \tag{5}$$

where $\mathbf{N}_\mathrm{q}$ and $\mathbf{N}_\mathrm{p}$ are respectively the shape function matrices of the displacement and pressure fields, and $\mathbf{q}$ and $\mathbf{p}$ the displacement and pressure unknowns nodal vectors.

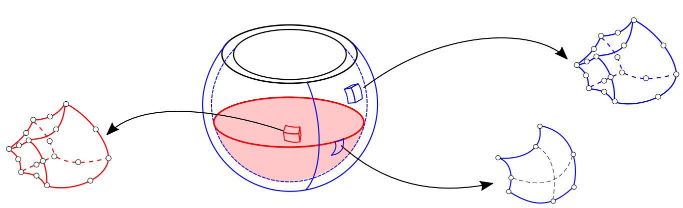

Figure 3: FE discretization with twenty nodes quadratic volumetric element (HEXA 20) for the solid and fluid domains and height nodes quadratic surfacic elements (QUAD 20) for surfacic domains. The fluid and the solid domains have coincident meshes.

2.2 Harmonic problem

From the $(\mathbf{u}, p)$ formulation we obtain the following harmonic system of equations:

$$\left(\mathbf{K} - \omega^2\mathbf{M}\right)\mathbf{q} + \mathbf{C}\mathbf{p} = \mathbf{F} \tag{6}$$

$$\mathbf{H}\mathbf{p} + \omega^2\mathbf{C}^{\mathrm{T}}\mathbf{q} = \mathbf{0} \tag{7}$$

where $\mathbf{K}$ the linear stiffness matrix, $\mathbf{M}$ the solid mass matrix, $\mathbf{F}$ the nodal external force vector, $\mathbf{H}$ the matrix associated with the bilinear pressure gradient and $\mathbf{C}$ the coupling matrix between the nodal displacements and nodal pressure vectors. In matrix form, the previous system can be written

$$\left(\begin{bmatrix}\mathbf{K} & \mathbf{C}\\ \mathbf{O} & \mathbf{H}\end{bmatrix} - \omega^2\begin{bmatrix}\mathbf{M} & \mathbf{O}\\ -\rho_{\mathrm{f}}\mathbf{C}^T & \mathbf{O}\end{bmatrix}\right)\begin{pmatrix}\mathbf{q}\\ \mathbf{p}\end{pmatrix} = \begin{pmatrix}\mathbf{F}\\ \mathbf{0}\end{pmatrix} \tag{8}$$

The associated eigenvalue problem is not symmetric and could generate numerical difficulties with classical algorithms. Note that the use of a $(\mathbf{u}, \varphi)$ formulation, based on a fluid potential displacement φ, can be used to symetrized the problem (see [2] for details). According to the pressure fluctuation condition on the free surface given by Equation (2), the matrix $\mathbf{H}$ is invertible and the pressure unknowns can thus be condensed on the solid displacements unknowns

$$\left[\mathbf{K} - \omega^2\left(\mathbf{M} + \mathbf{M}_{\mathrm{a}}\right)\right]\mathbf{q} = \mathbf{F} \tag{9}$$

where $\mathbf{M}_{\mathrm{a}}$ is the so called added mass matrix operator given by

$$\mathbf{M}_{\mathrm{a}} = \rho_{\mathrm{f}}\mathbf{C}\mathbf{H}^{-1}\mathbf{C}^T \tag{10}$$

The added mass matrix is symmetric positive definite but full. Its evaluation can leads to some numerical issues out of the scope of this paper.

2.3 Effect of the fluid height on the natural frequencies

The eigenvalue problem associated with Equation (9) is written as

$$\left[\mathbf{K} - \omega^2\left(\mathbf{M} + \mathbf{M}_{\mathrm{a}}\right)\right]\mathbf{X} = \mathbf{0} \tag{11}$$

where $(\omega_i^2, \mathbf{X}_i)$ for $i = 1 \ldots k$ are the first k eigenvalues and eigenvectors with $\omega_1 < \omega_i < \omega_k$. Note that in linear dynamics, when the fluid height increase during a filling process, the natural frequencies decrease due to the added mass effect [2].

3 HYDROELASTICITY AROUND A PRESTRESSED STATE

Let's consider the hydroelastic problem around a prestressed state defined in Figure 4. The total nodal displacement is given by $\mathbf{q}_{\mathrm{t}} = \mathbf{q}_{\mathrm{s}} + \mathbf{q}$ where $\mathbf{q}_{\mathrm{s}}$ is the solution of a non-linear quasi-static problem and $\mathbf{q}$ is a small amplitude displacement fluctuation. In the following, we assume that $\mathbf{q}_{\mathrm{s}}$ is already known from a quasi-static nonlinear simulation detailed in [1].

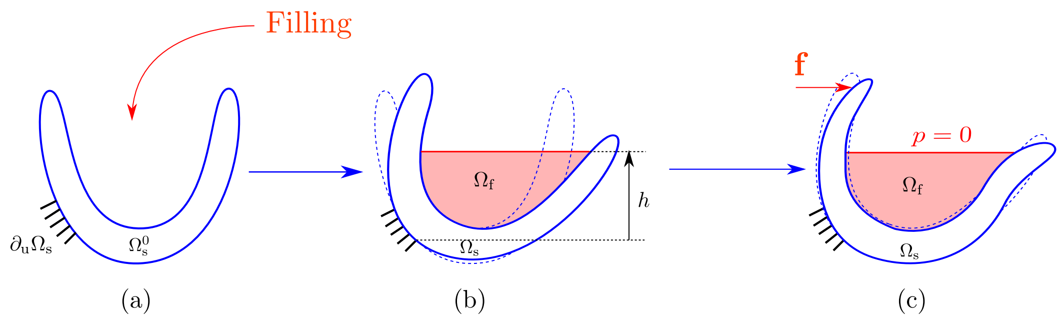

Figure 4: (a) Reference configuration without fluid; (b) currrent configuration for a given fluid height; (c) linearized vibration around a prestressed state under a small amplitude excitation **f**.

3.1 Discretized equations

Non-linear static equilibrium The discrete equilibrium equation of a partially filled elastic tank, with geometrical non-linearities, is given by

$$\mathbf{G}(\mathbf{q}_s) = \mathbf{F}_{\text{int}}(\mathbf{q}_s) - \mathbf{F}_{\text{ext}}(\mathbf{q}_s) = \mathbf{0} \tag{12}$$

where $\mathbf{F}_{\text{int}}$ and $\mathbf{F}_{\text{ext}}$ are respectively the internal and external nodal forces. The external forces correspond to the hydrostatic follower forces. Note that no volumetric fluid mesh is needed to solve the quasi-static non-linear equation [1].

Dynamic equilibrium For the discrete dynamic problem around the prestressed state, we assume that the total external force $\mathbf{F}_t$ is the sum of the hydrostatic follower force $\mathbf{F}_{\text{ext}}(\mathbf{q}_s)$ and a small external harmonic load $\mathbf{F}$. The discretized dynamic problem is thus defined by

$$\mathbf{G}(\mathbf{q}_s + \mathbf{q}) - \omega^2\mathbf{M}(\mathbf{q}_s)\mathbf{q} + \mathbf{C}(\mathbf{q}_s)\mathbf{p} = \mathbf{F} \tag{13}$$

$$\mathbf{H}\mathbf{p} + \omega^2\mathbf{C}^{\mathrm{T}}(\mathbf{q}_s)\mathbf{q} = \mathbf{0} \tag{14}$$

$\mathbf{M}(\mathbf{q}_s)$ and $\mathbf{C}(\mathbf{q}_s)$, which are the mass and coupling matrices defined on the current configuration of the solid, are noted $\mathbf{M}$ and $\mathbf{C}$ in the rest of the paper. Note that contrary to the quasi-static problem, meshes are needed for both fluid and solid volumetric domains in order to generate the discretized operators.

Linearization If the amplitude of the displacement fluctuation $\mathbf{q}$ is small compared to the quasi-static displacement solution $\mathbf{q}_s$, we obtain, by linearization and using Eq. (12), the following relation

$$\mathbf{G}(\mathbf{q}_s + \mathbf{q}) \simeq \frac{\partial \mathbf{G}(\mathbf{q}_s)}{\partial \mathbf{q}}\mathbf{q} = \mathbf{K}_{\tan}\mathbf{q} \tag{15}$$

where $\mathbf{K}_{\tan}$ is the tangent stiffness matrix. This tangent stiffness matrix is given by $\mathbf{K}_{\tan} = \mathbf{K}_{\text{mat}} + \mathbf{K}_{\text{geo}} - \mathbf{K}_{\text{fol}}$ where $\mathbf{K}_{\text{mat}}$ is the material tangent stiffness, $\mathbf{K}_{\text{geo}}$ the geometrical tangent stiffness and $\mathbf{K}_{\text{fol}}$ the follower tangent stiffness.

Consequently, Eq. (13) and (14) gives the linearized hydroelastic problem

$$\left(\mathbf{K}_{\tan} - \omega^2\mathbf{M}\right)\mathbf{q} + \mathbf{C}\mathbf{p} = \mathbf{F} \tag{16}$$

$$\mathbf{H}\mathbf{p} + \omega^2\mathbf{C}^{\mathrm{T}}\mathbf{q} = 0 \tag{17}$$

which can be written in matrix form

$$\left(\begin{bmatrix}\mathbf{K}_{\tan} & \mathbf{C}\\ \mathbf{O} & \mathbf{H}\end{bmatrix} - \omega^2\begin{bmatrix}\mathbf{M} & \mathbf{O}\\ \rho_{\mathrm{f}}\mathbf{C}^T & \mathbf{O}\end{bmatrix}\right)\begin{pmatrix}\mathbf{q}\\ \mathbf{p}\end{pmatrix} = \begin{pmatrix}\mathbf{F}\\ \mathbf{0}\end{pmatrix} \tag{18}$$

3.2 Effect of fluid height on natural frequencies

Using the approach detailed in the previous section, we obtain the following eigenvalue problem

$$\left[\mathbf{K}_{\tan} - \omega^2\left(\mathbf{M} + \mathbf{M}_{\mathrm{a}}\right)\right]\mathbf{X} = \mathbf{0} \tag{19}$$

Due to the fact that all the matrix operators depend on the non-linear quasi-static solution, we are not able to predict *a priori* the eigenvalue evolution, i.e. without solving the problem for each fluid height.

4 NUMERICAL EXAMPLE

In this section, we perform a parametric study to estimate the influence of the non-linearities on the dynamic response of an hydroelastic problem. The goal is to compare the added-mass and prestress effects on the eigenvalue problem given by the equation (19). Let's consider a cylinder with rigid walls and an elastic bottom, partially filled with liquid (Figure 5). The influence of the fluid height is studied and our numerical results are compared to the experimental ones given in [4].

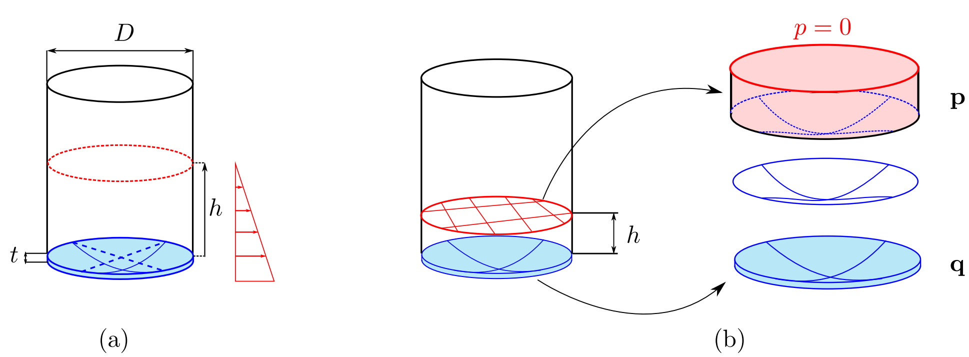

Figure 5: (a) Filling of a cylinder of diameter $D = 357$ mm with water $\rho_{\mathrm{f}} = 1000$ kg.m^{-1}, rigid walls and an elastic bottom of thickness $t = 0.357$ mm made of plexiglas. The material parameters are the Young Modulus $E = 5.47e9$ Pa and a Poisson Ration of 0.38 and the plexiglas density $\rho_{\mathrm{s}} = 1.38$ kg.m^{-3}. The fluid height value is such that $h \in [0, 250]$ mm; (b) Hydroelastic vibrations around the pre-stressed state for a given fluid height h.

To compute the natural frequencies of the coupled system, we proceed in two steps :

- Firstly, we compute the non-linear quasi-static equilibrium state of the rigid cylinder with a flexible bottom loaded by the hydrostatic follower forces for various fluid heights [1].
- Then, we remesh the fluid domain in order to obtain a coincident mesh between the fluid and the solid. This allows us to solve the eigenvalue problems (19) for each fluid height.

On Figure 6 we can see the evolution of first eigenfrequencies in terms of the fluid height (between 0 and 250 mm). Except for the first eigenmode, we can observe first a decrease then an increase of the natural frequencies during the filling process. The decreasing is due to the influence of the added-mass effect and the increasing is due to the prestress effect. Indeed, the added mass effect is related to the kinetic energy of the displaced fluid by the elastic bottom leading to a decrease of the frequencies. On the contrary, the prestress effect leads to a stiffening of the structure.

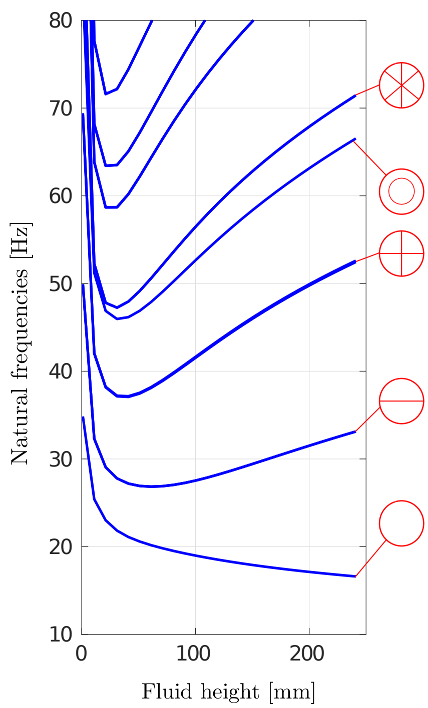

Figure 6: Evolution of natural frequencies in function of the fluid height with $\mathbf{K}_{\text{tan}} = \mathbf{K}_{\text{mat}} + \mathbf{K}_{\text{geo}} - \mathbf{K}_{\text{fol}}$

The order of magnitude of the computed natural frequencies is very close to the experimental observations from [4]. This validate our numercial approach.

5 CONCLUSION

In this paper, a Finite Element approach has been proposed to compute the dynamic behaviour of an elastic tank partially filled with fluid. Both the added-mass and the prestress effects on the linearized hydroelastic vibrations are included in the model. Numerical results are obtained for a cylinder with rigid walls and an elastic bottom, partially filled with liquid. These results are in very good agreements with the experimental observations from [4]. Indeed, the added mass effect decreases the natural frequencies contrary to the pre-stress effect which increases them. The extension to more complex geometries will be the object of further investigations.

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