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NUMERICAL SIMULATION OF LOW CYCLE FATIGUE OF STRUCTURAL MATERIAL WITH INCLUSIONS

JOURY M. TEMIS, KHAKIM KH. AZMETOV AND ALEKSEY A. LAZAREV

Central Institute of Aviation Motors (CIAM)
2, Aviamotornaya St., Moscow, Russia
e-mail: tejoum@ciam.ru, web page: http://www.ciam.ru

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Abstract. A model of cyclic stress-strain curve and a corresponding damage model are developed for a simulation of cyclic loading and estimating of the life time of structure materials with inclusions. Material cyclic model describes the change of Bauschinger's effect, Young's module and parameters of hardening as functions of accumulative plastic strain. The damage model is based on reaching the ultimate state of accumulative plastic strain at cyclic loading. Simulation of the cyclic plasticity of structure materials with inclusions is based on an original finite elements code. Results of numerical tests of structural materials with different inclusions are presented.

1 INTRODUCTION

Structural materials with inclusions are found and used in different constructions. On the one hand, introduction of inclusions of various types into a base material will greatly enhance mechanical properties of the resultant composite one. However, these properties will depend on the properties of both the base and the inclusion materials used. It is also important to take into consideration the ratio of inclusion volumes and their geometrical shapes. Together with the parameters of the interface between the base material and inclusions, they define fatigue strength under cyclic loading. The main problem that the material designer faces is creation of composite materials and structures with superior mechanical properties. To solve this problem, experimental research is mainly employed. This method is expensive and it does not allow an easy optimisation of material or component properties. The use of mathematical simulation methods appears to offer scope for focusing on the more likely material and inclusion combinations and thus limiting the amount of experimentation necessary to produce successful materials. Simultaneously, this method will allow more accurate predictions of mechanical behaviour of any components and structure that may go into production. On the other hand, the inclusion is a defect of the structural material and a location of potential cracks due to concentration of stress in the interface between the base material and inclusion. This stress-concentrated zone occurs around the location of inclusion.

2 THEORY

Successful prediction of low-cycle resource of the structural materials depends on the solution of several interrelated problems: i) development of an analytical model (loads, boundary conditions, materials model) that will adequately describe all aspects of the stress-strain history; ii) creation of a plasticity model that is adequate to processes taking place during elastoplastic deformation with alternating sign and iii) creation of criteria to define the life-time fatigue failure rate. The low-fatigue phenomenon is connected directly with the plastic deformation process in the stress concentration zones of structural materials. During the sign-alternating non-stationary non-elastic deformation the zone accumulates hidden damage, leading to appearance and evolution of cracks.

Simulation of low fatigue in structural materials is based on FEM and uses the following models: the model of cyclic plasticity of a material, the model of damage accumulation [1, 2], and the model of ñdyingò elements describing the evolution of cracks caused by LCF [2]. All of them are included in the original FE code for low-cycle fatigue simulation to analyse the behaviour and predict the lifetime of plastic materials with small rigid inclusions.

Consider cyclic deformation where a load vector $\{F\}$ applies to a component in the following way:

0 Ÿ $\{F_1\}$ Ÿ $\{F_2\}$ Ÿ é Ÿ $\{F_k\}$ Ÿ é

If strain and stress vectors in body points {Ů}$_k$ and {ů}$_k$ correspond to an end of the k^{th} halfcycle of loading and {Ů}$_{k+1}$ and {ů}$_{k+1}$ and correspond to an end of the $(k+1)^{th}$ halfcycle then for each halfcycle we can use the following variational relationship

$$\int_{\Omega}\{\sigma\}_q^T\{\delta\varepsilon\}d\Omega-\int_{\Omega}\{F_{\Omega}\}_q^T\{\delta u\}d\Omega-\int_{S}\{F_S\}_q\{\delta u\}dS=0\,. \tag{1}$$

where $q = k,\ k + 1$ are halfcycle numbers. The nomenclature here and in the rest of the paper corresponds to the one used in paper [2].

Taking the difference between Eq. (1) at $q = k + 1$ and at $q = k$, we obtain that to model the stress-strain state occurring when a loading halfcycle changes to an unloading one, one needs to solve the following equation:

$$\int_{\Omega}\{\Delta\sigma\}_{k+1}^T\{\delta\varepsilon\}d\Omega-\int_{\Omega}\{\Delta F_{\Omega}\}_{k+1}^T\{\delta u\}d\Omega-\int_{S_F}\{\Delta F_S\}_{k+1}\{\delta u\}dS=0\,. \tag{2}$$

Having specified the functional form φů(φŮ), Eq.(2) can be reduced to the finite element problem using standard methods

$$[K]_{k+1}\cdot\{\Delta U_{k+1}\}=\{\Delta F_{k+1}\}. \tag{3}$$

where $[K]_{k+1}$ is the stiffness matrix of the $(k + 1)^{th}$ halfcycle, defined by step-by-step approach; {φU_{k+1}} and {φF_{k+1}} are vectors of increments of half-cycle displacements and loads, respectfully.

In this case the displacement vector at the $(k + 1)^{th}$ halfcycle is

$$\{U_{k+1}\}=\{U_k\}+\{\Delta U_{k+1}\}. \tag{4}$$

while strain and stress in Θcalculated point are related thus:

$$\begin{aligned}\{\varepsilon_{k+1}\}&=\{\varepsilon_k\}+\{\Delta\varepsilon_{k+1}\},\\ \{\sigma_{k+1}\}&=\{\sigma_k\}+\{\Delta\sigma_{k+1}\}.\end{aligned} \tag{5}$$

Now let us introduce the unified stress-strain cyclic curve representation Fig.1. So, at the $(k + 1)^{th}$ halfcycle, the point representing the deformation process moves along the branch of the cyclic strain curve shown by the thick line originating from point ů$_k$ on Fig.1.

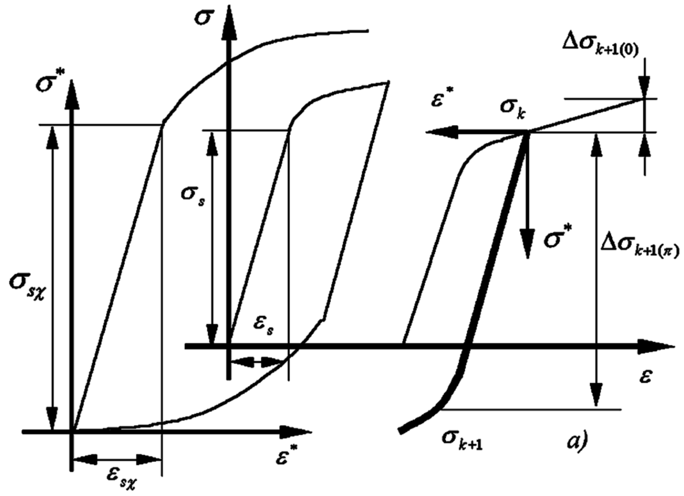


Figure 1: Strain curve definition for $(k + 1)^{th}$ halfcycle: a) strain curve branches subject to loading direction

The branch of the cyclic strain curve at every halfcycle is defined on the basis of a three-parametric model of material behaviour [1, 2], in accordance with the accumulated plastic strain $\chi = \sum_{n=0}^{n_f} \left|\Delta\varepsilon_p\right|$

$$\sigma^* = \begin{cases} E_\chi \varepsilon^* & \varepsilon^* \le \varepsilon^*_{s\chi} \\ E_\chi \varepsilon^*_{s\chi} + b_\chi \left[f\left(\varepsilon_s + \frac{\varepsilon^* - \varepsilon^*_{s\chi}}{b_\chi} \right) \right] & \varepsilon^* > \varepsilon^*_{s\chi} \end{cases}. \tag{6}$$

Here Ů$_{sG}$ = a_GŮ/ d_G, $d_G = E_G / E$; ů$_s$ and Ů$_s$ is stress and strain corresponding to start point of plastic deformation under monotonic loading; a_G, b_G and d_G are material parameters describing plastic deformation response of the material under cyclic loading; E_G is elastic unloading modulus, dependent on the amount of accumulated plastic strain χ ($E = E$G(G = 0)). The Bauschinger effect a_G may be defined as ů$_{sG}$ / ů$_s$ (Fig.1). A transformation coefficient b_G relates the non-linear portion of the stressï strain curve under the monotonic loading. The alternatingï sign plastic deformation tests conducted for different materials [1, 3, 4] showed that, for the constantï amplitude stress, constantï amplitude strain and randomï amplitude stress, the number of half-cycles n_f before failure is related to the limiting value χ_{max} by the power law

$$n_f = (\chi_{max}/\delta)^\gamma. \tag{7}$$

where δ is the constant depending on the residual plastic strain value, γ is the parameter that characterises the ability of material to ñcureò the cyclic loading damage.

At the same time, the accumulated plastic strain may be plotted on the co-ordinate plane (χ, n_f) using log-scale (Fig. 2). Moreover, if $\Delta\varepsilon_p$ does not change sign from half-cycle to half-cycle, then χ increases and n_f remains constant. If in the two adjoined half-cycles $\Delta\varepsilon_p$ changes sign, then n_f increases by one. Setting the measure of damage D = G(n) / G$_{max}$(n) to 1, we can define the amount of half-cycle loading where the alternatingï sign plastic deformation takes place.

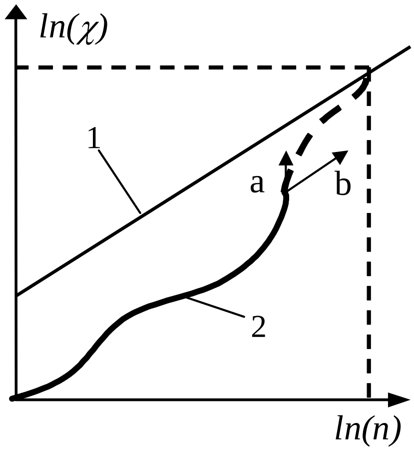


Figure 2: LCF life-time exhaustion process: 1 ï ultimate curve $\chi_{max}(n_f)$; 2 ï $\chi(n)$ change characteristics; a, b ï change direction of $\chi(n)$ depending on loading program

Once the value of D of a particular finite element reaches unity, the fatigue damage failure level of the material has been reached and $D = D_{\lim}$. The elastic modulus E of the element will be reduced (Fig.3). The effect of this is such that the contribution of the ñdyingò element on the stiffness of the overall system will be sharply reduced and a redistribution of stress and strain to the neighbouring elements will occur. Overall, some elements will experience a decrease in stress whilst others will become more stressed and, with increasing number of cycles, more ñdyingò elements will appear, causing further stress redistribution [2].

When an individual finite element has a value $D > D_{\lim}$, its stiffness matrix will be formed on the basis of parameter E_{crack} (Fig.4). It is assumed that the ñdyingò finite element is elastic and its elastic modulus in the stiffness matrix is found from the relationship

$$E_{crack} = \begin{cases} E_{\lim}, & \varepsilon_{cur} \le \sigma_{\lim} / E_{\lim} \\ \sigma_{\lim} / \varepsilon_{\lim}, & \sigma_{\lim} / E_{\lim} < \varepsilon_{cur} \end{cases} \tag{8}$$

and element stress is respectively found from relation

$$\sigma_{crack} = \begin{cases} \varepsilon_{cur} \cdot E_{\lim}, & \varepsilon_{cur} \le \sigma_{\lim} / E_{\lim} \\ \sigma_{\lim}, & \sigma_{\lim} / E_{\lim} < \varepsilon_{cur} \end{cases} \tag{9}$$

where ε_{cur} is the current element deformation, ů$_{\lim}$ the elements stress and $E_{\lim}$ the elements elastic modulus.

The models described above are included in original FE code and allow simulation of stress-strain kinetics and cyclic life exhaust process of the specimens and structures under different load conditions (rigid, soft, random and others). At that it is possible to estimate a number of cycles to crack origin from a condition of obtaining of failure rate to critical value D^* and to retrace a crack evolution process that crack path is simulated the ñdyingò elements.

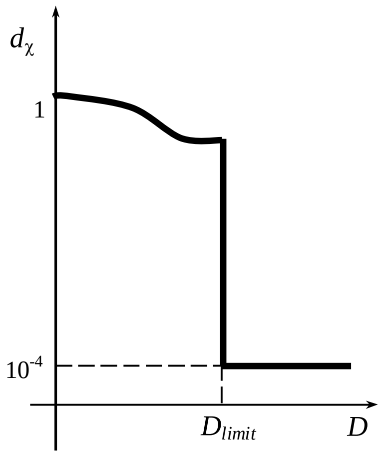


Figure 3: Change of parameter d_c for ñdiedò element

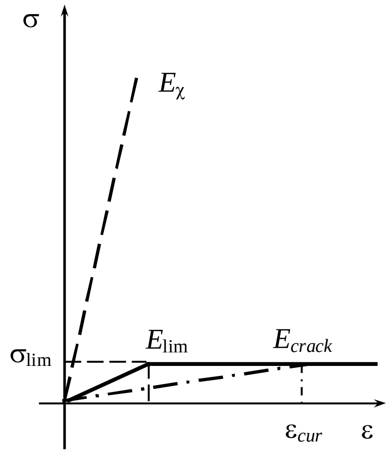


Figure 4: Stress-strain relation for ñdiedò element

3 SIMULATION RESULTS

In general, the stress-strain analysis of structure material with inclusions is a 3D plasticity problem. However, as a first approach [5, 6] the inclusions can be approximated as a spherical body surrounded by a cylinder of matrix material, as shown in Fig.5. The cylinder represents one cell of the inclusion. The dimensions of the cylinder are defined by condition that they are inscribed in a cuboid with height H and the base side $2R$ and all calculations were conducted for the model using $H = 2R$.

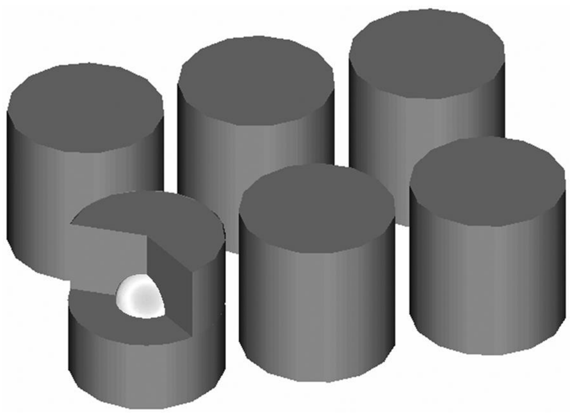


Figure 5: Schematic model of the structure material with spherical inclusions

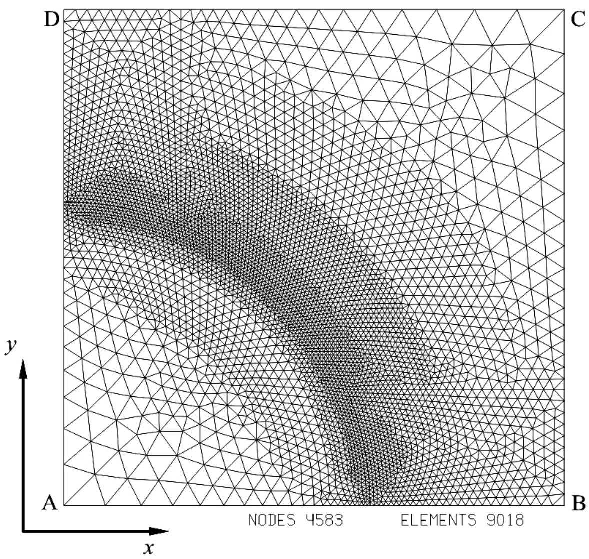
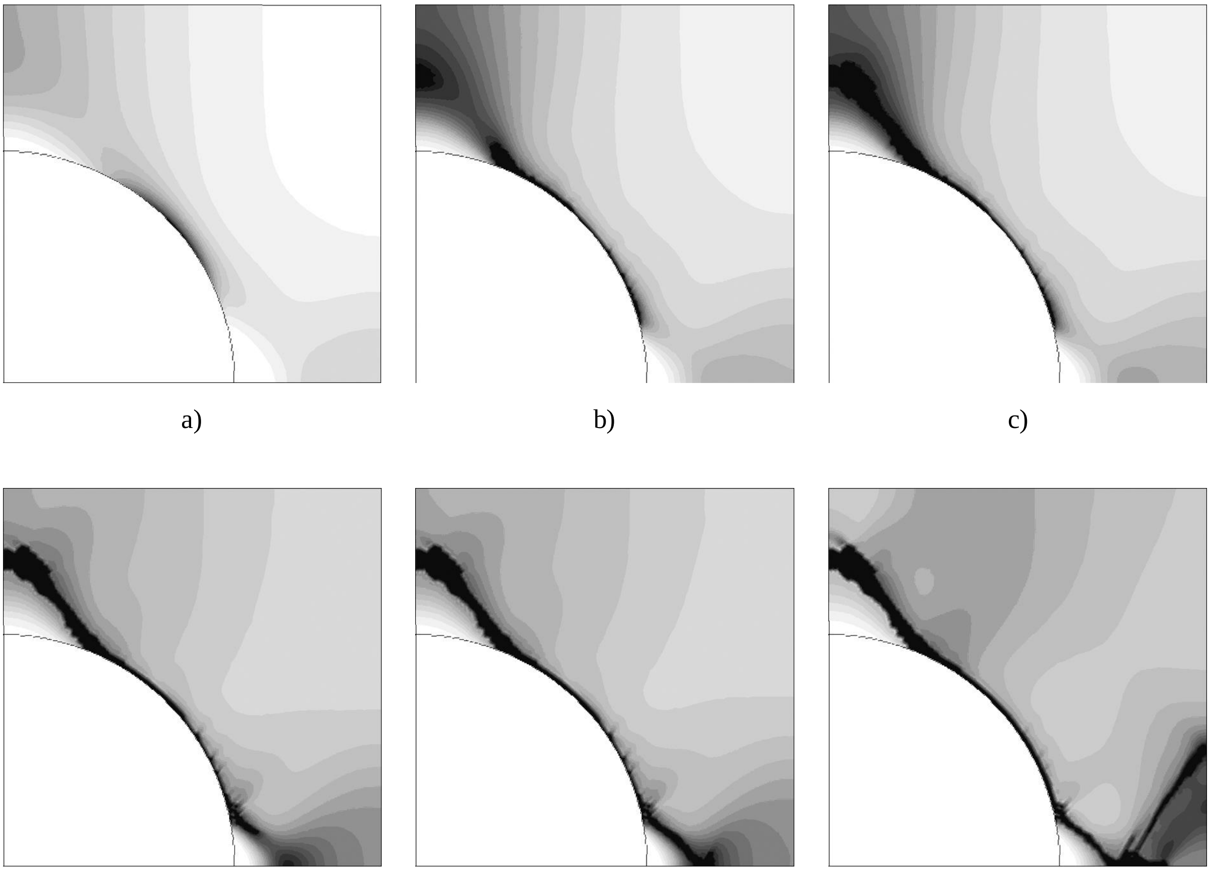


Figure 6: Finite element mesh for structure material with spherical inclusion

The FE model of axisymmetric simplex finite elements for structural material with spherical inclusion is shown on Fig.6. The main material is aluminium alloy and the inclusion is silicon carbide.

The numerical simulation of the structural material with spherical inclusions was carried out in strain-controlled tests for a number of strain amplitudes of symmetric cycles. At each calculation point the current values of χ and damage measure D were defined. Calculation results using ñdyingò elements technique were shown on Fig.7. LCF crack initiates at an interface zone (Fig.7a), than the crack grows along the interface and another crack appears at the front zone (Fig.7b); then the interface and front cracks merge (Fig.7c). Then the crack moves toward the middle zone (Fig.7d), the interface-frontal and middle crack merge (Fig.7e). At last, the crack exits to the outside of the specimen and a full destruction of material takes place (Fig.7f).

This FE calculation up to the full destruction, compared with the predicted results using Theory of Cells (TOC) [5-7] have been plotted as Strain / Cycle Number curve in Fig.8.



a) b) c)

d) e) f)

Figure 7: LCF crack growth: a) crack initiation at interface zone; b) interface crack growth and crack initiation at frontal zone; c) interface and frontal crack merging; d) crack initiation at middle zone; e) interface-frontal and middle cracks merging; f) exit of crack to outside and full fracture of material

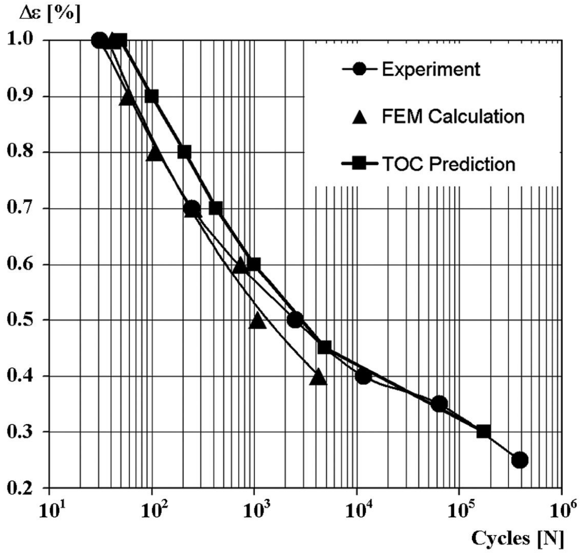


Figure 8: Fatigue diagram up to full fracture

Numerical simulations of the structural material with different inclusions were carried out in strain-controlled tests for a number of strain amplitudes of symmetric cycles. Calculation results with using ñdyingò elements technique were shown on Fig.9.

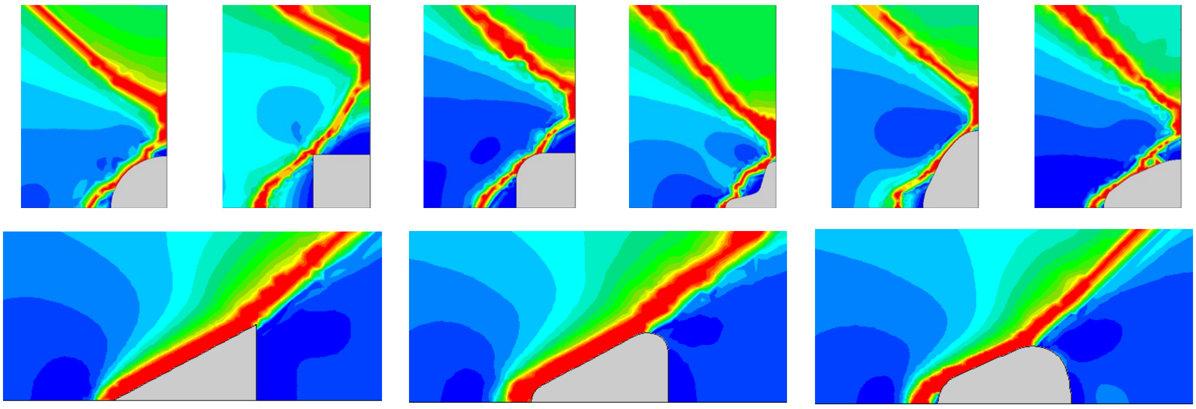


Figure 9: Crack path in structure materials with different inclusions

Simulation results showed that main crack growth behaviour depends on inclusion form symmetry. If inclusions are symmetric then the main crack grows symmetrically towards both sides from the inclusion (Fig.10a). Otherwise local cracks continue to grow to be major (Fig.10b).

Crack initiation and growth process is influenced by the correlation between the properties of the main material and the inclusion. If properties of the inclusion are greater than those of the main material, then a local crack arises at the interface between inclusion and main material (Fig.11a) and then transforms into a major crack (Fig.11b). Otherwise, local cracks arise and grow inside of the inclusion (Fig.12a), come out into the main material (Fig.12b) and transform into a major crack (Fig.12c).

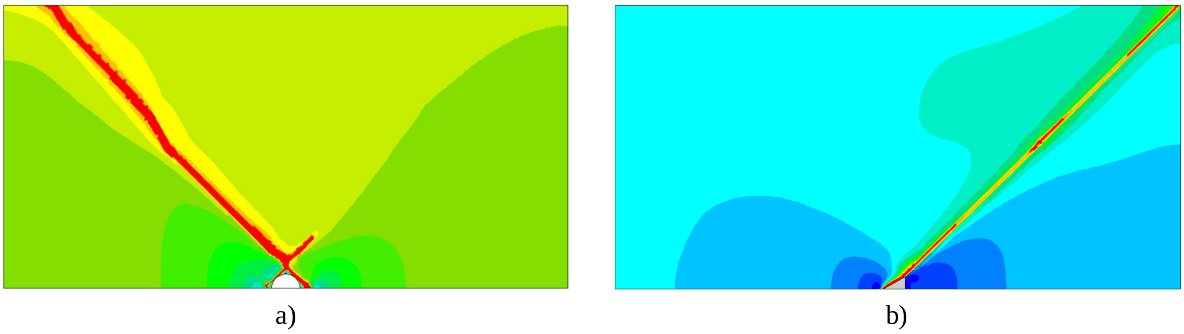


Figure 10: Main crack grow behaviour: a) symmetric inclusion; b) asymmetric inclusion

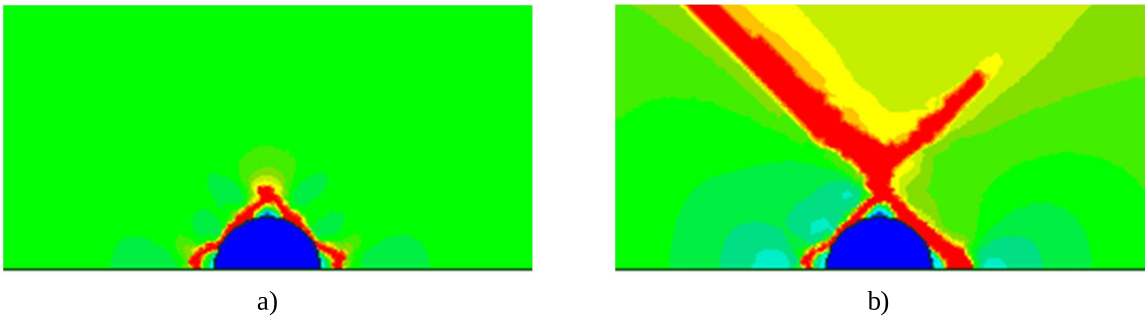


Figure 11: Crack growth process at inclusion properties greater main material ones

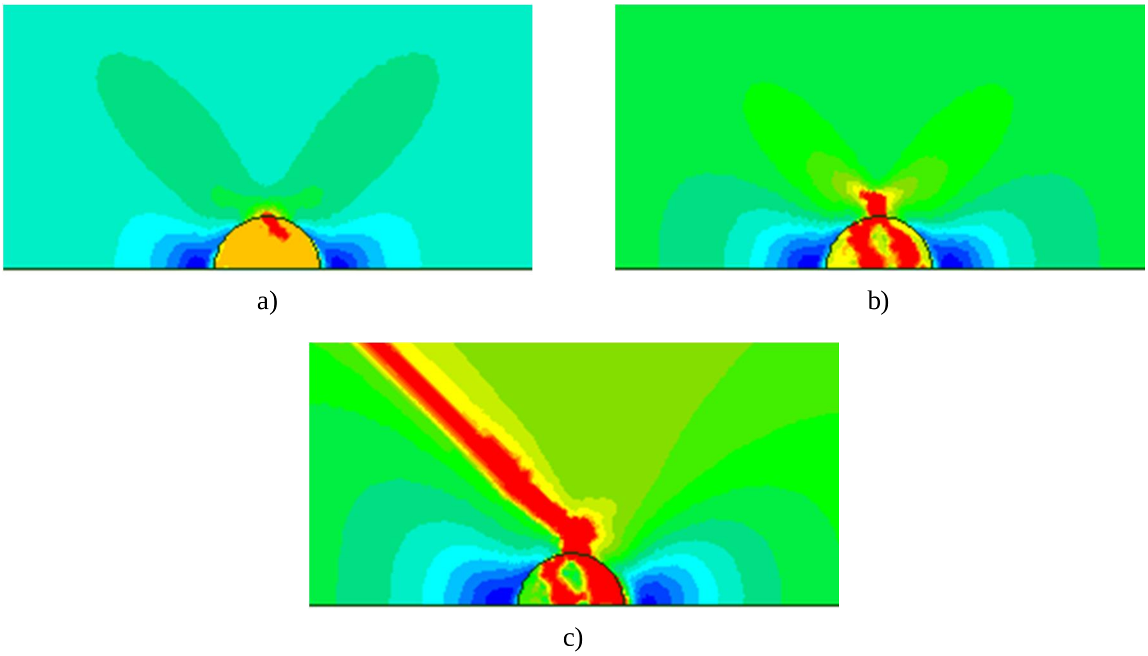


Figure 12: Crack growth process at inclusion properties less main material ones

A number of numerical tests of specimens with inclusions was carried out in order to obtain a relationship of lifetime cycle number by inclusion and main material properties ratio. Test results are presented on Fig. 13 where cycle number curves of local crack initiation and the full destruction of specimen are shown. The ratio of properties being equal to one indicates a specimen without an inclusion. Property ratio being equal to zero indicates that the inclusion is a void. If a property ratio is less one then inclusion material properties are less than those of the main material and vice versa (see Fig.11 and 12).

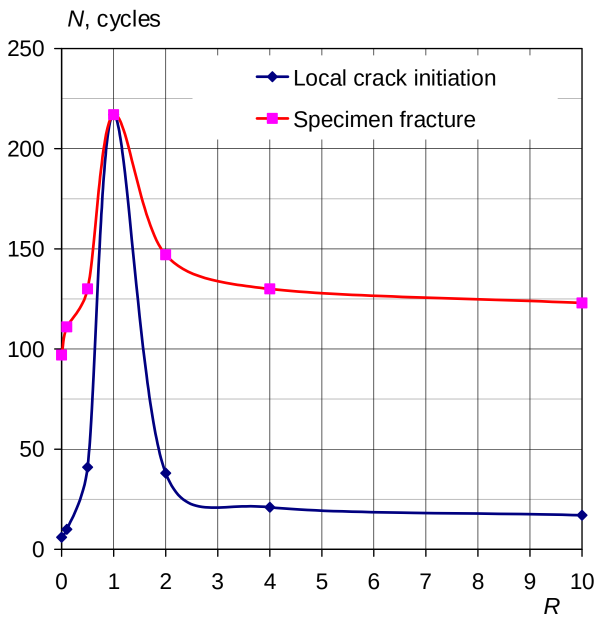


Figure 13: Specimen lifetime cycle number (*N*) in relation of main and inclusion materials properties ratio (*R*)

CONCLUSION

- A procedure for mathematical simulation the elastoplastic deformation processes in a structural material with inclusions under cyclic loading is presented. The results obtained show that failures of such materials occur in several stages.
- Crack initiation and growth behaviour depends on both the symmetry of the shape of the inclusion and the correlation between the main material and the material of the inclusion.
- Lifetime fatigue dependence on the ratio of properties of the inclusion and the main material is derived and plotted diagrammatically.

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