

ANALYTICAL DEFINITION OF THE CRITICAL LOAD OF A LONG-REINFORCED CONCRETE PIPE WITH VARIABLE GEOMETRY

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Abstract. This An analytical approach based on the Rayleigh method is adopted to calculate the buckling load of an actual 46-m-high concrete pipe, taking into account the geometric stiffness, functions of the concentrated forces, and self-weight of the structure. Stability analysis that included the self-weight of structural elements was originally discussed by Euler (1774), but he did not succeed in obtaining a satisfactory solution. The problem was finally resolved by Greenhill in 1881. The structural pipe analyzed in this work is made of concrete and its geometric nonlinear behavior and imperfections are linearized by reducing the structural stiffness. The material is considered to be viscoelastic with reduction of the flexural stiffness to consider the material non-linearity and creep is taken into account by the criteria from Eurocode 2. The ground is modeled as a set of distributed springs. Then, the critical buckling load is calculated analytically and dynamically defined to different instants along time after the structure to be loaded. Modulus of elastic and specific deformation on time are also obtained. Finally, the structural stiffness is evaluated. Reductions of 25.15% to the modulus of elasticity and deformation, and of 21.08% to the critical load of buckling for analysis performed at zero and four thousand days were found.

1 INTRODUCTION

Frame structures modeled as columns constitute the fundamental elements of various industrial applications. A column represents a continuous structural compression member whose vibrations are governed by nonlinear partial differential equations for which exact analytical solutions cannot be found [1]. The compressive capacity of a column can be quantified in various ways, including the conservative Euler load [2]. The elastic buckling load of a pin-ended column, an important design parameter of slender columns, was first determined by Euler in 1774 [3]. However, Euler found it difficult to include the effects of the self-weight of the column in its buckling response. Although the problem was finally solved by Greenhill in 1881 [4], the inclusion of the self-weight of a column has continued to be a topic of extensive mathematical discussion. Euler buckling, defined as the phenomenon in which an elastic member buckles under a sufficiently large compressive axial load, is perhaps the simplest and the most widespread type of column instability, the behavior of which has been confirmed by

Lubbers, van Hecke, and Coulais [5].

A continuous system as a column can readily be reduced to a single-degree-of-freedom (SDOF) system; thus, the modes in which the system will deform are restricted and the properties of the system can be expressed as functions of generalized coordinates. This technique was proposed by Rayleigh in his study of elastic system vibration, and his equations were found to be valid over the entire domain of the problem [6]. However, most actual structures are complex systems because their properties vary along their length. In such cases, the integrals obtained using the Rayleigh method can be solved within the limits established for each interval, i.e., the generalized properties can be calculated for each discrete segment of the structure, as defined by its geometry within that segment.

To analytically define the critical load of buckling for the case modeled in this study, all the elastic stiffness components are considered in the calculation, including the conventional stiffness, which depends on the material behavior, the geometric stiffness, which depends on the normal force acting on the structure, see [7] to [10], and the soil parcel, which accounts for the soil-structure interaction. It is important to note that the soil-structure interaction cannot be ignored, particularly in the case of a monopile foundation, because it may significantly influence the dynamic behavior of the structure [11]. Further, the influence of the normal force on the stiffness of unidimensional systems was examined by Wahrhaftig and Brasil, who studied a problem involving a significantly deformed beam [12].

The structure selected for this study is a slender reinforced concrete (RC) pipe for which the critical buckling load was calculated from a dynamic perspective. The nonlinearity of the material was computed by reducing its flexural stiffness, as seen in [13], reflecting the development cracking in the concrete, which are dependent on the magnitude of the stress.

Because the structure examined in the present study is extremely slender, the effects of creep significantly influence the calculated critical buckling load. Creep is a mechanical phenomenon that typically occurs in viscoelastic materials such as concrete, and it is an intrinsically nonlinear material property. The mathematical model for creep employed in the present study complies with the Eurocode criteria. Therefore, the primary nonlinearity considered is geometric, and it is solved by computing the second-order geometric stiffness parcel. The effects of material nonlinearity are captured by reducing the flexural stiffness, and the effects of creep are computed according to the Eurocodes [14].

2 LONG-REINFORCED CONCRETE PIPE WITH VARIABLE GEOMETRY

The problem evaluated in the present study involves calculating the critical buckling load of an actual slender reinforced concrete pipe with variable geometry that presents both geometrical and material nonlinearities (see Figure 1). The geometric details of the evaluated pipe are shown in Figure 2, where g denotes gravitational acceleration; Gr means ground; s represents each structural segment; S , D and th are the type, the external diameter, and the wall thickness of the section; d_b represents the reinforcing bar diameter; n_b is the number of reinforcing bars, and c' is the reinforcing cover.

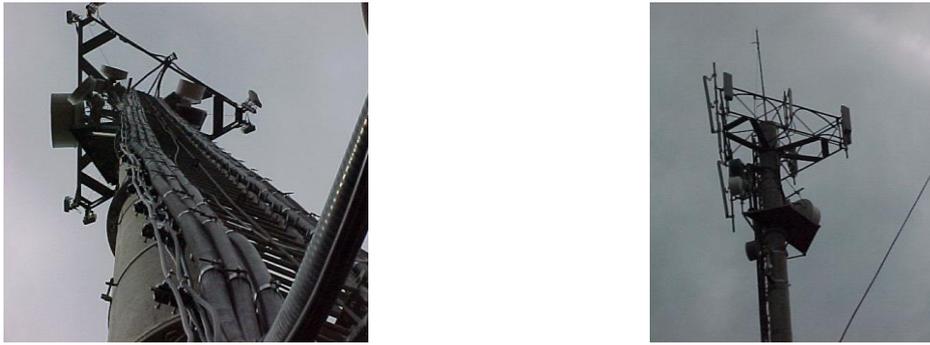


Figure 1. Photos of the tower

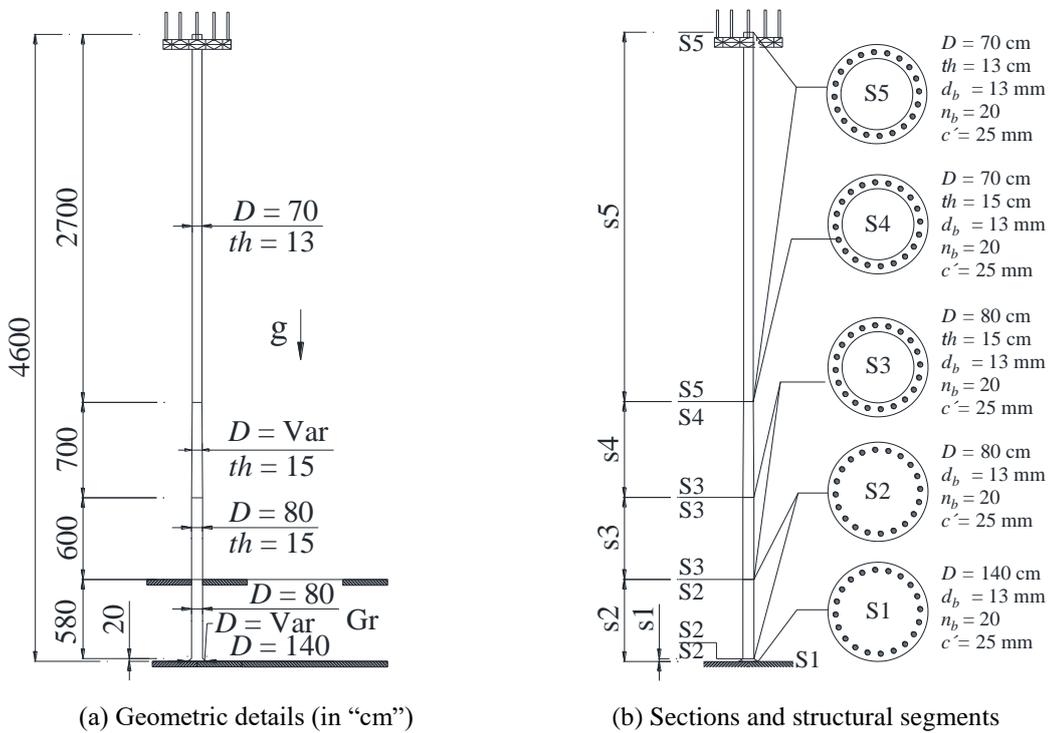


Figure 2. Subject reinforced concrete pipe

The structure is 46 m high; it has a hollow circular section. The slenderness ratio of the tower structure is 334. A set of antennae and a platform are installed at the tip of the structure, constituting a total mass of 1097.76 kg. Cables, a ladder, and a guardrail are installed along the entire length of the structure, adding a distributed mass of 40 kg/m. The density of the centrifuged RC pole was taken as 2600 kg/m³ and the density of the foundation shaft was 2500 kg/m³. The equipment installed on the tower is summarized in Table 1.

The foundation is a relatively deep shaft having a bell diameter of 140 cm, bell length of 20 cm, shaft diameter of 80 cm, and shaft length of 580 cm. The lateral soil resistance is represented by an elastic parameter, S_p , equal to 2668.93 kN/m³, providing a total lateral stiffness. The concrete resistance was taken as 30 MPa for the pole and 20 MPa for the

foundation. The physical nonlinearity of the RC was computed in accordance with the Brazilian Association for Standardization (ABNT, 2014), which suggests a 50% reduction in the gross moment of inertia [15].

Table 1. Devices, weights, and corresponding locations on the subject structure

Device	Height	Density / unitary mass / lumped mass
Pipe	6–46 m	2600 kg/m ³
Foundation	0–6 m	2500 kg/m ³
Distributed mass	6–46 m	40 kg/m
Lumped mass	46 m	1097.76 kg

Because this is an RC structure, it is necessary to account for the presence of the reinforcing bars when calculating the moment of inertia, which is accomplished by homogenizing the cross section. Therefore, according to the theorem of parallel axis, the factor F_{hs} , which multiplies the nominal moment of inertia of the section in terms of the total moment of inertia of the reinforcing steel, in the homogenized section is $F_{h1} = 1.0199$, $F_{h2} = 1.0568$, $F_{h3} = 1.0811$, $F_{h4} = 1.0671$, and $F_{h5} = 1.0859$.

3 CREEP CONSIDERATION

Creep, which represents a gradual increase in deformation over time, is a typical phenomenon noted in concrete structures owing to the viscoelastic nature of the concrete material. It is critical to consider creep in the verification of the stability of slender compressed members in the ultimate limit state because the stiffness of these members is modified as a function of the rheology of the material. Mathematically, creep can be characterized by models where the immediate elastic deformation is increased by viscous deformation, resulting in a temporal function for deformation. Consequently, the modulus of elasticity must also be provided as a temporal function that provides accurate results under normal levels of tension. Because of the viscous nature of concrete, even at a constant stress level, the deformation of a concrete structural element tends to increase over time.

The model that accounts for creep in EN 1992-1-1 [14] is based on the CEB-FIP Model Code [16]. In the method specified by the Eurocode standard, the effects of and variations in creep behavior over time are taken into account. The Eurocodes establish hypothetical and model limitations for the calculation of creep, in which the coefficient of creep, φ , is predicted as a function of the tangent modulus of elasticity, E_c . The creep deformation of concrete is obtained by multiplying the immediate deformation by the creep coefficient. The total concrete deformation at time t for a constant temperature is obtained by the sum of the terms for immediate deformation, creep, and shrinkage.

In the method specified by EN 1992-1-1 [14], all the factors related to the concrete, loading, and environmental characteristics are calculated as constant values for the studied time interval, and they comprise a single result for the coefficient of creep, φ . This coefficient must be directly introduced into the equation for slow deformation, and it serves as input data for various procedures. The basic equations for determining the creep coefficient of concrete over time use the average compressive strength, f_{cm} . The creep coefficient, $\varphi(t, t_0)$, as defined in Eq. (1).

$$\varphi(t, t_0) = \varphi_0 \beta_c(t, t_0) \quad (1)$$

The factor φ_0 , defined by Eq. (2) consists of three other factors. The first, φ_{RH} (given by Eq. (3) for average concrete compressive strength less than or equal to 35 MPa), accounts for the influence of the relative humidity of the environment, RH , and the equivalent thickness of the member, h_0 , which is a function of the cross-sectional area, A_c , and the external perimeter in contact with the environment, u_e , as defined in Eq. (4). The second factor, $\beta(f_{cm})$ (Eq. (5)), refers to the direct influence of resistance on φ_0 , and the third factor, $\beta(t_0)$ (Eq. (6)), refers to the age of the concrete, t_0 , when loading commences.

$$\varphi_0 = \varphi_{RH} \beta(f_{cm}) \beta(t_0) \quad (2)$$

$$\varphi_{RH} = 1 + \frac{1 - RH / 100}{0.1 \sqrt[3]{h_0}}, \text{ to } f_{cm} \leq 35 \text{ MPa} \quad (3)$$

$$h_0 = \frac{2A_c}{u_e} \quad (4)$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} \quad (5)$$

$$\beta(t_0) = \frac{1}{(0.1 + t_0^{0.20})} \quad (6)$$

The factor, $\beta_c(t, t_0)$ (Eq. (7)), considers a coefficient, β_H (Eq. (8) for average concrete compressive strength less than or equal to 35 MPa), to regulate the combined influence of relative humidity and equivalent thickness by simulating a real case in which the percolation path of adsorbed water in a robust section of concrete is so large that the effect of creep due to differential moisture is less important compared to that in the case of slimmer sections.

$$\beta_c(t, t_0) = \left[\frac{(t - t_0)}{\beta_H + (t - t_0)} \right]^{0.3} \quad (7)$$

$$\beta_H = 1.5 \left[1 + (0.012RH) \right]^{18} h_0 + 250 \leq 1500, \text{ to } f_{cm} \leq 35 \text{ MPa} \quad (8)$$

Therefore, the creep coefficient can be obtained using Eq. (1), and the temporal function describing deformation in accordance with EN 1992-1-1 [26] can be obtained as

$$\varepsilon(t, t_0) = \sigma_c(t_0) \left[\frac{1}{E_{ci}(t_0)} + \frac{\phi(t, t_0)}{E_{ci}(t_{28})} \right], \quad (9)$$

which leads to a function for the modulus of elasticity with respect to time as follows:

$$E(t, t_0) = \frac{1}{\frac{1}{E_{ci}(t_0)} + \frac{\phi(t, t_0)}{E_{ci}(t_{28})}}, \quad (10)$$

where $E_{ci}(t_0)$ is the modulus of elasticity when loading commences and $E_{ci}(t_{28})$ is the modulus of elasticity at 28 days.

4 ANALYTICAL DEFINITION OF THE CRITICAL BUCKLING LOAD

Consider the bar model shown in Figure 3, behaving in a non-damping free manner, to approximate the practical problem investigated in this study. The system is under the action of gravitational normal forces, originating from the distributed mass (the self-weight, ladder, cables, and guardrail) along the length of the tower, and the lumped mass at the tip (the antennae and platform). In addition, we consider the following well-known trigonometric function, which is assumed to be valid throughout the domain of the structure:

$$\phi(x) = 1 - \cos\left(\frac{\pi x}{2L}\right), \quad (11)$$

where x is the location of the calculation, originating at the base of the cantilever, and L is the length of the column, as shown in Figure 3, where t indicates dependency of the time, L is the length of the column, s is a segment, ϕ is a function, E is a modulus of elasticity, I is a moment of inertia, N is a normal force, m indicates mass, k is a stiffness, and S_o is a parameter related to the soil.

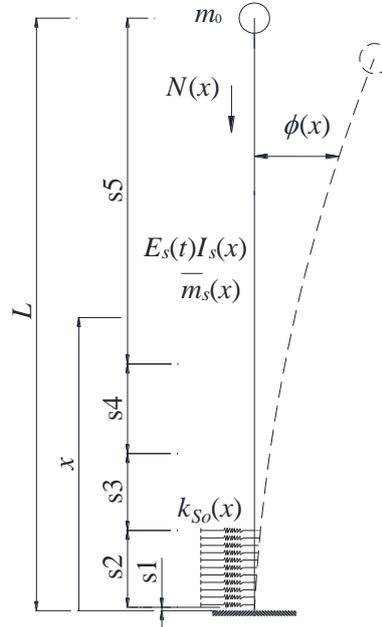


Figure 3. Frame element model in free vibration

This model represents a column under an axial compressive load, $N(x)$, with either constant

or variable properties along its length. These properties include the geometry, elasticity/viscoelasticity, and density, given by $I_s(x)$, $E_s(t)$, and $\bar{m}_s(x)$, respectively, where s denotes the considered segment. Applied springs of variable stiffness $k_{so}(x)$ act as the lateral soil resistance until the foundation elevation. To find the analytical solution of the present problem, it is necessary to consider the trigonometric function given by Eq. (11), the basic assumption of which, as a function of x , effectively restricts the bar to an SDOF system.

In the case of vibration of a cantilevered column that is fixed at its base but free at its tip, the shape function given in Eq. (11) satisfies the boundary conditions of the problem. The use of Eq. (11) as a shape function for an actual structure with varying geometry has been validated by Wahrhaftig [17]. This validation involved a comparison with a computational solution derived using FEM and other mathematical expressions.

By applying the principle of virtual work and its derivations, the dynamic properties of the subject system are obtained. The elastic/viscoelastic conventional stiffness is given by

$$k_{0s}(t) = \int_{L_{s-1}}^{L_s} E_s(t) I_s(x) \left(\frac{d^2 \phi(x)}{dx^2} \right)^2 dx, \text{ with } K_0(t) = \sum_{s=1}^n k_{0s}(t), \quad (12)$$

where for a segment s of the structure, $E_s(t)$ is the viscoelastic modulus of the material with respect to time; $I_s(x)$ is the variable moment of inertia of the section along the segment in relation to the considered movement, obtained by interpolation of the previous and following sections, already homogenized (if it is constant, it is simply I_s); $k_{0s}(t)$ is the temporal term for the stiffness; $K_0(t)$ is the final conventional stiffness varying over time; and n is the total number of segment intervals given by the structural geometry. In Eq. (12), obviously, t vanishes when the analysis considers a material with purely elastic, time-independent behavior.

The geometric stiffness appears as a function of the axial load, including the self-weight contribution, and is expressed as

$$k_{gs}(m_0) = \int_{L_{s-1}}^{L_s} \left[N_0(m_0) + \sum_{j=s+1}^n N_j + \bar{m}_s(x)(L_s - x)g \right] \left(\frac{d\phi(x)}{dx} \right)^2 dx, \text{ and} \quad (13)$$

$$K_g(m_0) = \sum_{s=1}^n k_{gs}(m_0), \quad (14)$$

where $k_{gs}(m_0)$ is the geometric stiffness in segment s , $K_g(m_0)$ is the total geometric stiffness of the structure with n as defined previously, and $N_0(m_0)$ is the concentrated force at the top, all of which are dependent on the mass m_0 at the tip. Further, N_j is the normal force from the upper segments, given by

$$N_0(m_0) = m_0 g \text{ and } N_j = \int_{L_{s-1}}^{L_s} \bar{m}_s(x) g dx, \quad (15)$$

where m_0 is the tip lumped mass at the element joint and $\bar{m}_s(x)$ is the mass per unit length defined. Then, the total generalized mass is given by

$$M(m_0) = m_0 + m, \quad (16)$$

considering that

$$m = \sum_{s=1}^n m_s, \text{ with } m_s = \int_{L_{s-1}}^{L_s} \bar{m}_s(x) (\phi(x))^2 dx, \text{ and } \bar{m}_s(x) = A_s(x) \rho_s, \quad (17)$$

where $\bar{m}_s(x)$ is the mass distributed to each segment s , which is obtained by multiplying the cross-sectional area, $A_s(x)$, by the density, ρ_s , of the material in the respective interval. Therefore, $\bar{m}_s(x)$ is the mass per unit length and m is the generalized mass of the system owing to the density of the material, with n as previously defined. If the cross section has a constant area over the interval, $A_s(x)$ will be just A_s ; consequently, the distributed mass will also be constant. Similarly, if the mass m_0 does not vary, all the other parameters that depend on it will also be constant.

One approach for considering the participation of the soil in the vibration of the system is to consider it as a series of vertically distributed springs that act as a restorative force on the system. With $k_{S_{os}}(x)$ denoting the spring parameter, the effective soil stiffness (as a function of the location x along the length) is generally defined as

$$K_{S_o} = \sum_{s=1}^n k_s, \text{ with } k_s = \int_{L_{s-1}}^{L_s} k_{S_{os}}(x) \phi(x)^2 dx, \text{ where } k_{S_{os}}(x) = S_{ops} D_s(x), \quad (18)$$

where the parameter K_{S_o} is an elastic characteristic consisting of the sum of $k_{S_{os}}(x)$ along the foundation depth $D_s(x)$, dependent on the geometry of the foundation, and the soil parameter S_{ops} . The soil parameter is considered to be (but need not necessarily be) constant in this case in each layer of soil. The variables s and n in Eq. (18) represent segments along the foundation and the total number of these segments, respectively. The parameter S_{ops} must be provided by a specialist geotechnical engineer, and different methods are used by different specialists for its determination. Note that K_{S_o} is obtained using Hamilton's principle (or the principle of virtual displacements) by considering the generalized distribution properties of the system, as in the case of the other stiffness and mass values. Further, note that the geometric parameters of the structure included in previous equations as a function of x are constant over their respective intervals. Considering the normal force as positive, we can obtain the total structural stiffness as

$$K(m_0, t) = K_0(t) - K_g(m_0) + K_{S_o}. \quad (19)$$

Finally, the natural frequency, as a function of the time and the mass at the tip, is calculated as

$$\omega(m_0, t) = \sqrt{\frac{K(m_0, t)}{M(m_0)}} (rd / s) \therefore f(m_0, t) = \frac{\omega(m_0, t)}{2\pi} (\text{Hertz}). \quad (20)$$

The mathematical procedure described above determines not only the frequency of a structure but also the critical buckling load. The criterion relevant to dynamic structures for determining the critical buckling load is established by assuming zero frequency at the moment when the structure loses its stiffness. As a result, the calculation expresses the load as a function of the mass. All the generalized parameters, such as the stiffness, $K = K(m_0)$ (Eq. (13)), normal force at the top, $N = N(m_0)$ (Eq. (15)), and mass, $M = M(m_0)$ (Eq. (16)), can be expressed a function of the mass at the top. Thus, the frequency can be written in terms of the mass at the

top as $f = f(m_0) = [K(m_0)/M(m_0)]^{1/2}$, the critical buckling load is reached when the frequency is zero. When introducing creep into the analysis, it is necessary to consider a model that represents the viscoelastic behavior of concrete. After the introduction of creep, the frequency and critical buckling load become temporal functions because the modulus of elasticity varies over time. Therefore, once creep is introduced into the calculation, the frequency can be written in terms of the time and the mass at the top as $f = f(m_0, t) = [K(m_0, t)/M(m_0)]^{1/2}$, and the resultant expression is sufficient to calculate the critical buckling load, determined when the frequency is zero at any arbitrary time after the structure is placed in service.

Taking all the previously explained in consideration, and making the mass m_0 at the top of the structural pipe to vary, the force acting at the top also varies, as does the frequency of the structure varies according to Eq.(20). Thus, the critical buckling load, N_{buck} , is defined at zero frequency as

$$N_0(m_0) = m_0 g, \text{ and for } f_c(t, m_0) = 0 \Rightarrow N_0(m_0)|_{f_c(t, m_0)=0} = N_{buck}. \quad (21)$$

The variation of the critical buckling load with the mass at the top of the structure can be observed in Figure 4, where the critical buckling load for the structure can be obtained for any stage in its lifetime.

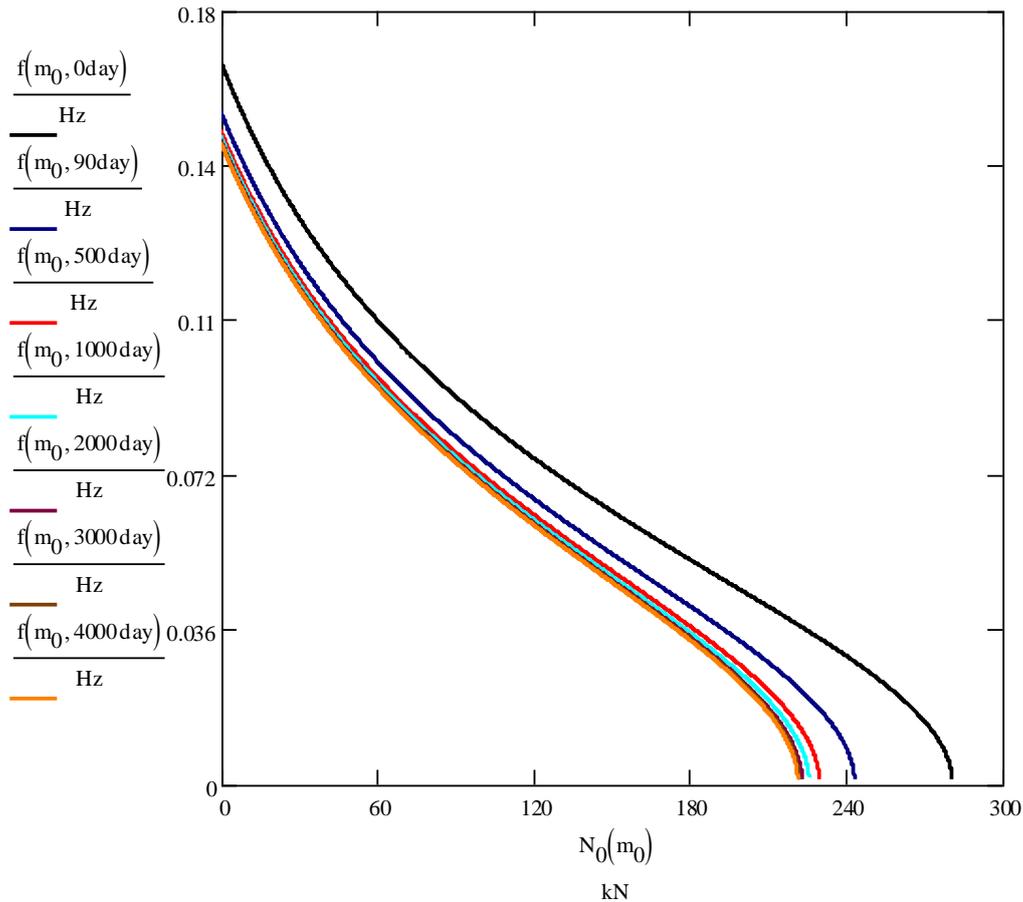


Figure 4. Critical buckling load over time

Table 2 presents the results obtained by the analytical procedure.

Table 2. Results of the proposed analytical procedure with respect to time.

Time (day)	Parameter by Eurocode		Analytical Procedure
	Modulus of elasticity (MPa)	Deformation (x 10 ⁻⁶)	Buckling load (kN)
0	18782.971	2.157	281.120
90	17050.349	2.566	243.551
500	14720.481	2.753	230.010
1000	14404.972	2.813	226.137
2000	14189.632	2.856	223.433
3000	14104.568	2.873	222.364
4000	14058.869	2.882	221.790
Variation	25.15%	25.15%	21.08%

12 CONCLUSIONS

- The critical buckling load was also obtained by the analytical process and was established as a compressive force of 281.120 kN at time zero and 221.790 kN at 4000 days, both corresponding to the nullity of the initial first natural frequency, what represents a reduction of 21.08%.
- Because of the creep, the modulus of elasticity and specific deformation have had both a variation of 25.15% along 4000 days.
- Directions for future research include the application of the proposed analytical method to other structures. Further, it remains necessary to verify the effects of creep on the dynamic behavior of structural models at scale.

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