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A NONLINEAR A-PRIORI HYPER-REDUCTION METHOD FOR THE DYNAMIC SIMULATION OF A CAR TIRE ROLLING OVER A ROUGH ROAD SURFACE

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Abstract. An a-priori hyper-reduction method for nonlinear structural dynamics finite element problems is proposed in this work. A-priori calculated static configurations and eigenmodes are used rather than time-domain training simulation data to create a reduced order basis and perform the hyper-reduction element selection. The hyper-reduction element selection is performed by solving an L_1 optimization problem subject to a set of equality constraints. The proposed method is applied to the case of a highly nonlinear high-fidelity tire finite element model rolling with a constant angular velocity over a rough road surface. It is shown that care has to be taken during the hyper-reduction process when considering distributed contact constraints, as is the case for e.g. a tire rolling over a rough road surface. Large speedup factors can be obtained while still retaining a relatively high accuracy, making the proposed method suitable for application to e.g. design optimization.

1 INTRODUCTION

When optimizing the design of a passenger car tire, typically over 50 different performance criteria have to be taken into account. These are related to (but not limited to) e.g. energy efficiency, handling, wear and noise. Due to the complex structure of a typical passenger car tire, most of the criteria are coupled: trying to enhance one performance will often decrease other performances. In order to cope with the increasing need to optimize multiple tire performance criteria simultaneously, predictive numerical simulation techniques could be used, rather than time-consuming experiments. A numerical approach using high-fidelity nonlinear structural dynamics finite element (FE) tire models offers the possibility to perform various virtual studies and virtual assessment of different tire designs. Because of the too high numerical cost, currently no industrially applicable high-fidelity fully-predictive numerical approach is available to model a tire rolling over a rough road surface. Such a numerical approach is necessary to predict and assess e.g. the airborne exterior noise and structure-borne interior noise performance of a tire design, amongst others. In order to alleviate the computational cost associated with the use of the high-fidelity FE models, the principle of projection-based nonlinear model order reduction (MOR) [3] can be applied. The original full order model (FOM) is transformed into a smaller, lower dimensional reduced order model (ROM) by means of projecting the full order solution space onto a lower dimensional subspace. As demonstrated in e.g. [4] and [7], significant speedups can be achieved using nonlinear MOR, therefore re-enabling the use of highly complex high-fidelity FE models. The choice of the subspace basis determines how good the FOM solution is approximated by the ROM solution. Several approaches have been suggested for nonlinear structural dynamics applications: (i) Proper Orthogonal Decomposition (POD) applied to a series of dynamic training snapshots [3], [7], [8]. (ii) Modal Derivatives (MD) in combination with e.g. eigenmodes [10], [14], [16]. (iii) Linear or nonlinear static configurations in combination with e.g. eigenmodes [4], [5], [9], [12], [15]. While the POD approach can offer a very efficient low-dimensional subspace basis, relevant simulation training data is required. Computation of the training data can be too expensive, especially in high-fidelity predictive approaches as used for tire design optimization. The MD approach has been shown to be usable for structural dynamics problems with mild nonlinear behavior of the internal force term [14], [16]. Therefore, the static configuration approach is adopted in this work to create the subspace basis. Recently, so-called hyper-reduction methods [7], [12] for nonlinear structural FE problems have been proposed. These hyper-reduction methods are a class of projection-based nonlinear MOR methods designed specifically for the reduction of nonlinear structural FE models. The projection of the FOM onto a subspace is complemented by a second-tier approximation, which is a necessary step to effectively reduce the computational complexity of the ROM [7]. The Energy Conserving Sampling and Weighting (ECSW) method proposed by Farhat et al. [7] has been shown to preserve the structure, symmetry and stability properties of the FOM [8]. The ECSW method [7] requires expensive time-domain training simulations, which have to be performed sequentially as each consecutive timestep depends on the previous one. For large FOMs, as used in tire design optimization problems, this time-domain training approach is too expensive to use, as per design change the time-domain training has to be repeated. Therefore, there is a clear need for a-priori methods which do not need time-domain training data. A hyper-reduction method is proposed in this work that uses a-priori static training simulations, similar to the Multi-Expansion Point Modal (MEM) method [12] and the static ECSW training method proposed by Rutzmoser et al. [13]. These a-priori static training simulations can be performed in parallel, therefore making the hyper-reduction method suitable even for the predictive simulations in tire design optimization. The proposed hyper-reduction method is applied to a nonlinear high-fidelity FE tire model rolling with a constant angular velocity over a rough road surface. The predicted vertical contact forces and general tire response due to the rolling are of interest for the prediction of the airborne exterior noise and structure-borne interior noise performance of the considered tire design. Throughout this work, the rolling of the tire over the rough road surface with a constant angular velocity is referred to as the tire/road problem.

2 PROBLEM STATEMENT

The full order model (FOM) of the tire/road problem is described by the semi-discrete nonlinear equations of motion as:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}_{\mathbf{ALE}}\left(\mathbf{x}\right)\dot{\mathbf{x}} + \mathbf{f}\left(\mathbf{x}, \dot{\mathbf{x}}\right) - \mathbf{f}_{\mathbf{ALE}}\left(\mathbf{x}\right) = \mathbf{f}_{\mathbf{p}}\left(\mathbf{x}, p_a\right) + \mathbf{f}_{\mathbf{c}}\left(\mathbf{x}, \mathbf{x}_{\mathbf{r}}\right) + \mathbf{f}_{\mathbf{e}} \tag{1}$$

Here $\mathbf{M} \in \mathbb{R}^{n\times n}$ is the configuration-independent mass matrix, $\mathbf{G}_{\mathbf{ALE}}\left(\mathbf{x}\right) \in \mathbb{R}^{n\times n}$ is the ALE skew-symmetric Gyroscopic matrix and $\mathbf{f}_{\mathbf{ALE}}\left(\mathbf{x}\right) \in \mathbb{R}^{n}$ the ALE rotational inertia force vector [11] which describe the constant rolling, $\mathbf{f}\left(\mathbf{x}, \dot{\mathbf{x}}\right) \in \mathbb{R}^{n}$ the internal force vector which includes nonlinear strain-displacement behavior and nonlinear nearly-incompressible (visco-)hyperelastic constitutive behavior, $\mathbf{f}_{\mathbf{p}}\left(\mathbf{x}, p_a\right) \in \mathbb{R}^{n}$ the air-pressure force vector and p_a the applied inflation air pressure, $\mathbf{f}_{\mathbf{c}}\left(\mathbf{x}, \mathbf{x}_{\mathbf{r}}\right) \in \mathbb{R}^{n}$ the tire/road contact force vector which is described using a penalty method formulation, $\mathbf{x}_{\mathbf{r}}$ the current road surface contact constraints and $\mathbf{f}_{\mathbf{e}} \in \mathbb{R}^{n}$ the configuration-independent external force vector. The current configuration is defined as $\mathbf{x} = \mathbf{x}_{\mathbf{0}} + \mathbf{u} \in \mathbb{R}^{n}$, where $\mathbf{x}_{\mathbf{0}}$ is the reference configuration and $\mathbf{u}$ the total displacement at time t with respect to the reference configuration. The time dependency is omitted from notation for clarity. The first and second derivatives of the current configuration with respect to time are denoted as $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$ respectively. In order to describe the nearly-incompressible behavior of the (visco-)hyperelastic constitutive models, a mixed displacement-pressure (u/p) formulation is used [2]. A variant of the implicit generalized-α discrete time integration method, proposed by Arnold and Brüls [1], is chosen to time-discretize the equations of motion (1). This variant of the generalized-α method has second-order accuracy for the acceleration field variables. Theory and implementation details can be found in [1]. An implicit rather than an explicit time integration scheme is chosen, as this allows to use larger timesteps. This results in less timesteps to be evaluated and in general lower overall computational costs. As an implicit scheme is used, the nonlinear equations of motion (1) are consistently linearized in the spatial domain around the current configuration $\mathbf{x}$. This yields the following set of equations:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{G}_{\mathbf{ALE}}\dot{\mathbf{x}} + \left(\mathbf{K} - \mathbf{K}_{\mathbf{p}} - \mathbf{K}_{\mathbf{c}} - \mathbf{K}_{\mathbf{ALE}}\right)\boldsymbol{\Delta}\mathbf{u} = \mathbf{f}_{\mathbf{p}} + \mathbf{f}_{\mathbf{c}} + \mathbf{f}_{\mathbf{e}} - \left(\mathbf{f} - \mathbf{f}_{\mathbf{ALE}}\right) \tag{2}$$

Here $\mathbf{K} \in \mathbb{R}^{n\times n}$ is the internal force tangent stiffness matrix, $\mathbf{K}_{\mathbf{p}} \in \mathbb{R}^{n\times n}$ is the pressure load force tangent stiffness matrix, $\mathbf{K}_{\mathbf{c}} \in \mathbb{R}^{n\times n}$ is the contact force tangent stiffness matrix and $\mathbf{K}_{\mathbf{ALE}} \in \mathbb{R}^{n\times n}$ is the ALE inertia matrix [11]. Solving the set of linearized equations of motions (2) per iteration step quickly becomes very costly as the amount of degrees of freedom rises. For industrial-sized design problems, e.g. the tire/road problem discussed in this work, the total computational cost becomes too large and the simulations are no longer feasible to use in a design context. Therefore, hyper-reduction is applied to the FOM (2) to create the hyper-reduced order model (HROM).

3 REDUCED ORDER MODEL DEFINITION

The full-order solution $\mathbf{x} \in \mathbb{R}^n$, residing in the solution manifold defined by (1), is approximated by the reduced-order solution $\mathbf{q} \in \mathbb{R}^m$, which resides in a constant subspace of the solution manifold. The subspace is spanned by the columns of $\mathbf{V} \in \mathbb{R}^{n \times m}$, hereafter called the reduced order basis (ROB). This leads to the following approximation:

$$\mathbf{x} \approx \tilde{\mathbf{x}} = \mathbf{x_0} + \mathbf{V}\mathbf{q} \tag{3}$$

And since $\mathbf{V}$ is chosen to be constant, it follows that:

$$\dot{\mathbf{x}} \approx \dot{\tilde{\mathbf{x}}} = \mathbf{V}\dot{\mathbf{q}} \tag{4}$$
$$\ddot{\mathbf{x}} \approx \ddot{\tilde{\mathbf{x}}} = \mathbf{V}\ddot{\mathbf{q}} \tag{5}$$

When inserting the approximations (3)–(5) in the linearized equations of motion (2), this introduces an error which can be eliminated by the Galerkin projection of the linearized equations of motion (2) onto the ROB $\mathbf{V}$, resulting in the ROM:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \tilde{\mathbf{G}}\dot{\mathbf{q}} + \tilde{\mathbf{K}}\boldsymbol{\Delta}\mathbf{q} = \tilde{\mathbf{f}} \tag{6}$$

Where

$$\begin{aligned}
\tilde{\mathbf{M}} &= \mathbf{V}^T\mathbf{M}\mathbf{V} \in \mathbb{R}^{m \times m} \\
\tilde{\mathbf{G}} &= \mathbf{V}^T\mathbf{G_{ALE}}\mathbf{V} \in \mathbb{R}^{m \times m} \\
\tilde{\mathbf{K}} &= \mathbf{V}^T\left(\mathbf{K} - \mathbf{K_p} - \mathbf{K_c} - \mathbf{K_{ALE}}\right)\mathbf{V} \in \mathbb{R}^{m \times m} \\
\tilde{\mathbf{f}} &= \mathbf{V}^T\left(\mathbf{f_p} + \mathbf{f_c} + \mathbf{f_e} - \mathbf{f}\right) \in \mathbb{R}^{m}
\end{aligned}$$

Both the reduced mass matrix $\tilde{\mathbf{M}}$ and reduced external force $\mathbf{V}^T\mathbf{f_e}$ can be calculated a-priori, as they are configuration independent and remain constant. The other reduced terms are configuration-dependent and therefore have to be evaluated again per iteration step. A second-tier hyper-reduction approximation of these terms is used to reduce the computational cost associated with their evaluation.

3.1 REDUCED-ORDER BASIS DEFINITION

Highly nonlinear behavior occurs in the tire/road problem due to the constitutive behavior of the rubber compounds and reinforcements, as well as the contact forces. Therefore the ROB as used in this work is generated using nonlinear steady-state (which are in fact static due to the ALE formulation [11]) contact configurations $\boldsymbol{\chi} \in \mathbb{R}^{n \times n_\chi}$ and eigenmodes $\boldsymbol{\Psi} \in \mathbb{R}^{n \times n_\Psi}$, where $n_\chi + n_\Psi = m$:

$$\mathbf{V} = \begin{bmatrix} \boldsymbol{\chi}^1 - \mathbf{x_0} & \cdots & \boldsymbol{\chi}^{n_\chi} - \mathbf{x_0} & | & \boldsymbol{\Psi}^1 & \cdots & \boldsymbol{\Psi}^{n_\Psi} \end{bmatrix} \in \mathbb{R}^{n \times m} \tag{7}$$

This ROB can be calculated a-priori and in parallel. A set of road surface constraints $\left[\mathbf{x_r}^1 \quad \cdots \quad \mathbf{x_r}^{n_\chi}\right]$ is sampled from the set of a-priori known road surface constraints. The corresponding steady-state (static) contact configurations $\boldsymbol{\chi}^k$, $k = 1 \cdots n_\chi$ are calculated as follows:

$$\mathbf{f}\left(\boldsymbol{\chi}^k\right) - \mathbf{f_{ALE}}\left(\boldsymbol{\chi}^k\right) = \mathbf{f_p}\left(\boldsymbol{\chi}^k, p_a\right) + \mathbf{f_c}\left(\boldsymbol{\chi}^k, \mathbf{x_r}^k\right) + \mathbf{f_e} \tag{8}$$

These contact configurations can be considered as *nonlinear* constraint modes, where a distributed set of constraints (the geometrical contact constraints) is applied rather than local unit displacements (as is the case for linear constraint modes [6]). Following the MEM method [12], sets of eigenmodes are calculated around nonlinear constraint modes $\boldsymbol{\chi}^l \in \boldsymbol{\chi}$, $l = 1 \cdots n_l \leq n_\chi$:

$$\boldsymbol{\Psi}_{\boldsymbol{\chi}^\mathbf{l}} = \mathrm{eig}\left(\bar{\bar{\mathbf{K}}}\left(\boldsymbol{\chi}^l\right), \mathbf{M}\right) \in \mathbb{R}^{n \times n_\Psi} \tag{9}$$

Here $\bar{\bar{\mathbf{K}}} = \mathbf{K} - \mathbf{K_p} - \mathbf{K_c} - \mathbf{K_{ALE}}$. The sets of eigenmodes are then concatenated:

$$\boldsymbol{\Psi}_{\boldsymbol{\chi}} = \left[\boldsymbol{\Psi}_{\boldsymbol{\chi}^\mathbf{1}} \quad \cdots \boldsymbol{\Psi}_{\boldsymbol{\chi}^{\mathbf{n_l}}}\right] \tag{10}$$

And a singular value decomposition is performed on the concatenated eigenspace, where the n_Ψ most dominant contributions are kept:

$$\mathbf{U\Sigma W}^T = \mathrm{svd}\left(\boldsymbol{\Psi}_{\boldsymbol{\chi}}\right) \tag{11}$$

$$\boldsymbol{\Psi} = \{\mathbf{U}_i | i = 1 \cdots n_\Psi\} \tag{12}$$

4 HYPER-REDUCED ORDER MODEL DEFINITION

Following the finite element assembly procedure, the reduced, configuration dependent terms in (6) can be written as:

$$\tilde{\mathbf{f}}\left(\mathbf{x}\right) = \mathbf{V}^T\mathbf{f}\left(\mathbf{x}\right) = \sum_{i=1}^{|E|} \mathbf{V}_i^T\mathbf{f}_i\left(\mathbf{x}\right) \tag{13}$$

Here E is the total set of elements and $\mathbf{V}_i^T$ and $\mathbf{f}_i$ have a sparse structure, only containing the contributions of element i. These reduced terms can be approximated further by using the Energy-Conserving Sampling and Weighting (ECSW) hyper-reduction approach as proposed by Farhat et al. [7]:

$$\tilde{\mathbf{f}}\left(\mathbf{x}\right) \approx \bar{\mathbf{f}}\left(\mathbf{x}\right) = \sum_{i=1}^{|E^s|} s_i \mathbf{V}_i^T\mathbf{f}_i\left(\mathbf{x}\right) \quad \text{and} \quad s_i > 0 \tag{14}$$

Here the reduced set of finite elements $E^s \subset E$ and their respective weights s_i are calculated (sampled from the full set E) using dynamic time-domain training data. Given a set of training samples $\mathbf{x}_i$, $i = 1 \cdots n_s$, the reduced set of elements E_s and their respective weights $\mathbf{s}$ are calculated in order to match the hyper-reduced and reduced internal forces.

The problem is written as a non-negative least squares (NNLS) optimization problem, and solved using a sparse NNLS solver as proposed by Farhat et al. [7]. As discussed by Farhat et al. [8] structure, stability and symmetry properties are preserved. Therefore, the ECSW Hyper-Reduction approach (14) is adopted in this work as well. The cost associated with the dynamic time-domain training simulations and solving the NNLS problem makes the ECSW element sampling method unfeasible for an a-priori approach. Recently, an alternative a-priori training approach was suggested by Rutzmoser et al. [13] that can be combined with the sparse NNLS solver [7]. Since in this work a set of nonlinear constraint modes is used to create the ROB, an alternative element sampling method is used that does not rely on the NNLS approach. More specifically, a variant of the Multi-Expansion point Modal Reduction method (MEM) proposed by Naets et al. [12] is used. The MEM method element sampling is performed by solving an L_1 optimization problem [12]. The hyper-reduced and reduced internal force and tangent stiffness matrix corresponding to one configuration are matched by means of equality constraints. Solving the L_1 optimization problems yields a set of elements E^s with a cardinality equal to $\frac{m^2+m}{2}$. The main benefit of this approach is that only one static training configuration needs to be considered, and an L_1 optimization problem solver of choice can be used. However, in case $\frac{m^2+m}{2} \geq |E|$, $E^s = E$ and no hyper-reduction is achieved. Due to distributed nature of the tire/road contact problem, a relatively large ROB is required and the MEM approach cannot be used. A variation of the MEM sampling approach, called the Multi-Configuration MEM (MCMEM) approach, is therefore suggested to cope with this issue. Instead of matching the hyper-reduced and reduced force and tangent stiffness corresponding to *one* configuration, the hyper-reduced and reduced forces (no tangent stiffness) corresponding to *multiple* configurations k are matched:

$$\begin{aligned} \min_{\mathbf{s}\in\mathbb{R}^{|E|}} \quad & |\mathbf{s}|_1 \\ \text{subject to} \quad & \bar{\mathbf{f}}\left(\mathbf{x_1}\right) = \tilde{\mathbf{f}}\left(\mathbf{x_1}\right) \\ & \vdots \\ & \bar{\mathbf{f}}\left(\mathbf{x_k}\right) = \tilde{\mathbf{f}}\left(\mathbf{x_k}\right) \\ & \mathbf{s} \geq 0. \end{aligned} \tag{15}$$

These constraints can be rewritten as:

$$\begin{bmatrix} \begin{bmatrix} \mathbf{V}_1^T \mathbf{f}_1\left(\mathbf{x}_1\right) \\ \cdots \\ \mathbf{V}_{|E|}^T \mathbf{f}_{|E|}\left(\mathbf{x}_1\right) \end{bmatrix}^T & \cdots & \begin{bmatrix} \mathbf{V}_1^T \mathbf{f}_1\left(\mathbf{x}_k\right) \\ \cdots \\ \mathbf{V}_{|E|}^T \mathbf{f}_{|E|}\left(\mathbf{x}_k\right) \end{bmatrix}^T \end{bmatrix} \mathbf{s} = \begin{bmatrix} \tilde{\mathbf{f}}\left(\mathbf{x}_1\right) \\ \vdots \\ \tilde{\mathbf{f}}\left(\mathbf{x}_k\right) \end{bmatrix} \tag{16}$$

$$\mathbf{A}_k^f \mathbf{s} = \begin{bmatrix} \tilde{\mathbf{f}}\left(\mathbf{x}_1\right) \\ \vdots \\ \tilde{\mathbf{f}}\left(\mathbf{x}_k\right) \end{bmatrix} \tag{17}$$

Solving this variation of the original MEM optimization problem yields a set E_s with a cardinality equal to $k \times m$. Therefore, as long as $k \times m \ll |E|$ and $\frac{m^2+m}{2} \geq |E|$, the

MCMEM approach can be used. Applying the MCMEM hyper-reduction to the linearized equations of motion (2) yields:

$$\tilde{\mathbf{M}}\ddot{\mathbf{q}} + \bar{\mathbf{G}}\dot{\mathbf{q}} + \bar{\mathbf{K}}\boldsymbol{\Delta}\mathbf{q} = \bar{\mathbf{f}} \tag{18}$$

Here the reduced and hyper-reduced terms are:

$$\begin{aligned}
\bar{\mathbf{G}} &= \sum_{i=1}^{|E^s|} s_i \mathbf{V}_i^T \mathbf{G}_{i,\mathbf{ALE}} \mathbf{V}_i \\
\bar{\mathbf{K}} &= \sum_{i=1}^{|E^s|} s_i \mathbf{V}_i^T \left(\mathbf{K}_i - \mathbf{K}_{i,\mathbf{ALE}}\right) \mathbf{V}_i - \sum_{i=1}^{|E_p^s|} s_{i,\mathbf{p}} \mathbf{V}_i^T \mathbf{K}_{i,\mathbf{p}} \mathbf{V}_i - \mathbf{V}^T \mathbf{K}_{\mathbf{c}} \mathbf{V} \\
\bar{\mathbf{f}} &= \sum_{i=1}^{|E_p^s|} s_{i,\mathbf{p}} \mathbf{V}_i^T \mathbf{f}_{i,\mathbf{p}} + \mathbf{V}^T \mathbf{f}_{\mathbf{c}} + \mathbf{V}^T \mathbf{f}_{\mathbf{e}} - \sum_{i=1}^{|E^s|} s_i \mathbf{V}_i^T \left(\mathbf{f}_i - \mathbf{f}_{i,\mathbf{ALE}}\right)
\end{aligned}$$

While all of the elements E are used to perform the MCMEM sampling of the internal and ALE force terms and to assemble the hyper-reduced form $\sum_{i=1}^{|E^s|} s_i \mathbf{V}_i^T \left(\mathbf{f}_i - \mathbf{f}_{i,\mathbf{ALE}}\right)$, a subset of elements, $E_p \subset E$ is used to perform the MCMEM sampling of the pressure force term and assemble to hyper-reduced form $\sum_{i=1}^{|E_p^s|} s_{i,\mathbf{p}} \mathbf{V}_i^T \mathbf{f}_{i,\mathbf{p}}$. This subset E_p corresponds to the elements to which the internal air pressure is applied.

4.1 REDUCED CONTACT FORCE CONSIDERATIONS

For the specific case of the tire/road problem, the subset of contact elements is small and no active contact search has to be performed. No benefit is achieved hyper-reducing the contact force term $\mathbf{f}_{\mathbf{c}}$, thus this term is re-projected per iteration step. Additional care has to be taken when evaluating the reduced contact force term. Due to the projection of the FOM on the ROB, the resulting ROM can lock when applying the contact constraints. This is a direct consequence of the reduction of the problem and the choice of the ROB, which is aggravated in the case of the tire/road problem due to: (i) The nature of the distributed contact problem, where many constraints are active at the same time. The tire configuration needs to comply with the road surface to meet the contact constraints locally, requiring local flexibility which is typically lost after the projection step. (ii) The hyper-elastic, nearly incompressible constitutive behavior in combination with a mixed u/p formulation is susceptible to locking [2], even without projection of the FOM on a ROB. The possible ROM solutions are constrained to lie in the reduction space, and it could be that enforcing the contact constraints results in configurations that do not lie in this space. Therefore, it is suggested to weaken the enforcement of the contact constraints by using a smaller penalty factor ϵ when evaluating the reduced contact force term. A smaller penalty factor allows a larger violation of the contact constraints, and can be seen as introducing local "flexibility" to prevent the locking of the ROM model by using a soft rather than hard enforcement of the constraints.

5 NUMERICAL VALIDATION AND RESULTS

The proposed MCMEM method is implemented in a nonlinear finite element Matlab framework. A nonlinear high-fidelity FE tire model consisting of 75400 elements and 326771 degrees of freedom is used as the FOM. Nonlinear visco-hyperelastic and hyperelastic constitutive models are used to model the rubber compounds and reinforcement materials. The tire is mounted on a rigid rim and inflated to a specific air pressure p_a. The numerical test-setup consists of the tire being loaded force-controlled on a rotating drum, on which the rough road surface is mounted. The external loading force is held constant. Compliance of the test-rig assembly (to which the tire is mounted) is included as a linear spring connected to the rigid rim center. The FOM and a set of road surface contact constraints is shown in Figure 1:

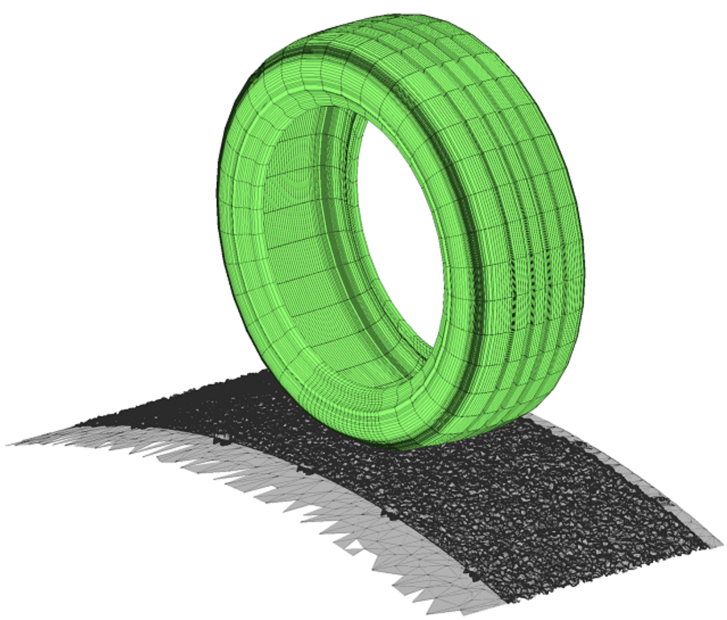


Figure 1: FOM and set of a road surface contact constraints

The drum rotates at an angular velocity corresponding to a surface velocity of $13.89\frac{\mathrm{m}}{\mathrm{s}}$ and drives the rotation of the tire. The tire can thus be considered to be in a free-rolling regime. The applied road surface contact constraints correspond to a maximum excitation frequency of 500 Hz. The corresponding minimal sampling frequency is 1 kHz, leading to a minimal timestep $\Delta t = 0.001s$. As discussed before, the hyper-reduced discretized equations of motion (18) are solved using a generalized-α integrator [1]. A smaller timestep is used to ensure stability, as the unconditional stability property does not hold for nonlinear problems [1]. A total of 0.1 seconds of constant rolling is simulated, using a discrete timestep $\Delta t = 0.0001s$. The relative error on displacements is defined as

$$\varepsilon_x = \frac{|\mathbf{x} - \tilde{\mathbf{x}}|_2}{|\mathbf{x}|_2}$$

and the relative error on the total vertical contact force is defined as

$$\varepsilon_{f_c} = \frac{|\mathbf{f_{c,z}} - \tilde{\mathbf{f}}_{\mathbf{c,z}}|_1}{|\mathbf{f_{c,z}}|_1}$$

where the L_1 norm is used rather than the L_2 norm, as the total vertical contact force is of interest.

5.1 INFLUENCE OF NUMBER OF MCMEM SAMPLES

The influence of the amount of MCMEM samples k used for calculating E^s on ε_x and ε_{f_c} is shown in Figure 2. The same ROB is used for the different amount of samples. The mean displacement error, calculated over the simulated 0.1 seconds, stabilizes when two or more samples are used. Although not shown here, the amount of samples to calculate E^p already stabilizes from one sample on, which could be attributed to the relatively constant nature of the pressure force term. The RMS error on total vertical contact force improves when using more samples, but not as drastically as the mean error on displacements.

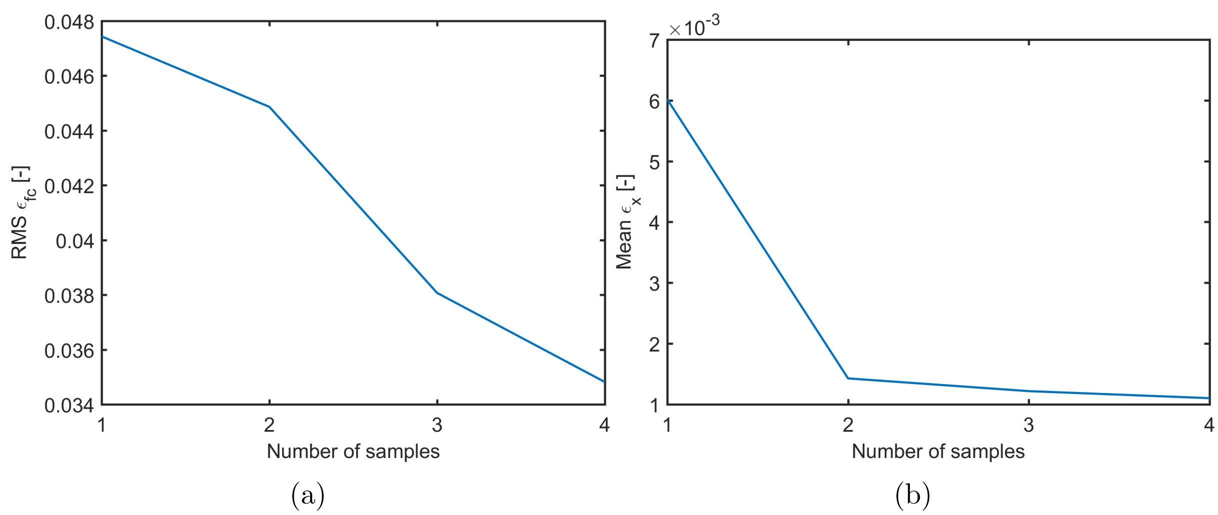
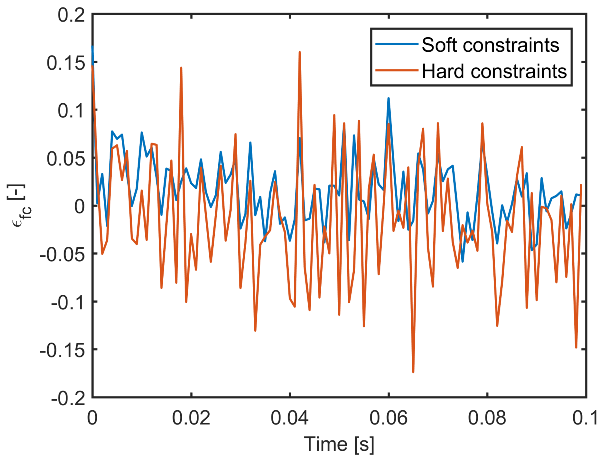


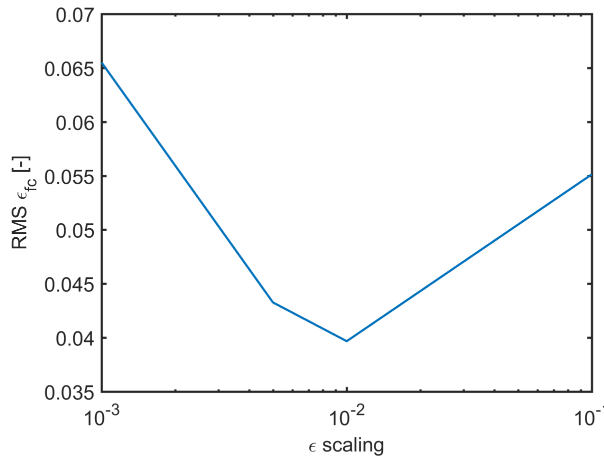
Figure 2: RMS relative error on total vertical contact force (a) and mean global relative error on displacements (b) for different amounts of MCMEM samples

5.2 INFLUENCE OF ENFORCING THE CONTACT CONSTRAINTS

Due to the distributed nature of the contact problem, locking of the HROM can occur when trying to enforce the original contact constraints, as discussed in section 4.1. The locking behavior can clearly be observed in the HROM contact forces. The effects of using a hard enforcement of the contact constraints versus a soft enforcement is shown in Figure 3 (a). An optimal value of the scaling of the penalty factor ϵ (as used for the soft constraint enforcement) can be calculated by means of comparing results for the case of e.g. a static loading, which is shown in Figure 3 (b). The same amount of MCMEM samples and same ROB are used for the comparison.



(a)



(b)

Figure 3: Relative error on total vertical contact force (a) and RMS relative error on total vertical force (b) for different scalings of the penalty factor ϵ

5.3 GLOBAL PERFORMANCE

The global performance of the MCMEM method is compared for different HROMs. These HROMS differ in the amount of nonlinear constraint modes and eigenmodes used for ROB construction. The same amount of MCMEM samples is used to calculate E^s and E_p^s for all HROMs. The same penalty scaling factor is used for all HROMs as well. An overview is given in Table 1. The ratio of the number of nonlinear constraint modes versus number of eigenmodes is kept constant for all HROMS. The online speedup factor is denoted as OSF (the time-domain simulation speedup), while the *total* speedup factor is denoted as TSF. It should be noted that TSF includes a fixed ROB calculation cost, which means that the TSF will tend to the OSF for longer simulated times. In general, applying the MCMEM method to the tire/road problem allows for good speedups while still retaining a relatively high accuracy, as demonstrated by the results shown in Table 1.

Table 1: Global performance of the MCMEM method for different HROMs

	# DOFs	$\|E\|$	$\|E_p\|$	$\varepsilon_{x,\text{mean}}$	$\varepsilon_{f_{c,\text{RMS}}}$	OSF	TSF
FOM	326771	75400	6552	-	-	-	-
HROM1	175	350	175	0.001947	0.05719	336.56	13.81
HROM2	350	700	350	0.001599	0.04929	159.72	13.21
HROM3	525	1050	525	0.001519	0.04947	93.54	12.48
HROM4	700	1400	700	0.001428	0.04486	59.36	11.59

6 CONCLUSION

An a priori nonlinear hyper-reduction method for nonlinear structural dynamics FE problems, such as the tire/road problem, is proposed in this work. The proposed MCMEM method is a variant of the MEM method [12], where hyper-reduced and reduced internal forces corresponding to multiple configurations are matched rather than matching the hyper-reduced and reduced internal force and tangent stiffness matrix corresponding to one configuration. The proposed MCMEM method uses a constant ROB consisting of nonlinear steady-state contact configurations and eigenmodes. It is shown that care has to be taken when including distributed contact constraints in the HROM, as locking of the HROM can occur. When using a penalty method to enforce the constraints, lowering the penalty factor alleviates this problem. Application of the MCMEM method to the highly nonlinear tire/road problem shows that large speedups can be achieved, while still retaining a relatively high accuracy. Future research will focus on the determination of an optimal set of nonlinear constraint modes and eigenmodes for the ROB and minimal amount of necessary MCMEM samples, as well as application of the MCMEM method to multi-physical problems.

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