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QUASI-TRIVIAL SOLUTIONS FOR UNCOUPLED, HOMOGENEOUS AND QUASI-HOMOGENEOUS LAMINATES WITH HIGH NUMBER OF PLIES

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Abstract. Quasi-trivial (QT) sequences are a class of lamination stacks for which, in the framework of Classical Laminate Theory (CLT), the properties of uncoupling and/or homogeneity are verified in a closed-form solution [1]. These sequences have received great attention from the scientific community as they have proved to be an extremely powerful tool for the design and optimization of composite laminates.

Nevertheless, two main reasons limit their adoption: first, to find QT sequences, a complex algorithm is required; second, calculations become computationally intensive for long QT sequences, thus limiting the maximum number of plies attainable. This constrains the use of QT stacks to applications involving only thin laminates.

In order to exploit QT stacks for thick laminates new tools are proposed. Firstly, a new and more efficient algorithm for finding QT stacking sequences is developed and an original procedure is devised to effectively code it. The proposed algorithm finds a greater number of QT solutions, with respect to those given in [1]. Additionally, analytical relationships to obtain new QT sequences by superposition of known QT sequences are presented in [2]. Thanks to this new class of closed-form solutions, laminates can be designed using QT stacking sequences without limitations on the maximum number of plies.

The results presented in this work open new possibilities for the design and optimisation of thick laminates. In addition, laminates with special requirements may be designed by superposition of QT stacks, thus reaching specific design goals that cannot otherwise be met.

1 INTRODUCTION

Advanced composite materials are extensively used in high-end applications, thanks to their extraordinary specific properties. However, part of this success is also due to the possibility they offer of being tailored to the specific application required. This allows overcoming some limitations related to metallic materials, but at the same time makes the design process quite cumbersome.

When dealing with the design of composite structures, laminates with identical plies are often used. The design of such structures involves both geometric variables and laminate stack parameters. These latter are the number of plies and their orientation angles. A major difficulty during design is anisotropy, which affects material behaviour at mesoscopic and macroscopic scales. In this regard, engineers often use simplifying hypotheses/rules to get some desired properties (e.g. symmetric stacks to get membrane/bending uncoupling, balanced ones to get membrane orthotropy, etc.). Unluckily, these design rules drastically reduce the design space and may lead to cut out classes of stacks that could potentially be optimum solutions for the problem at hand.

In this context, the introduction of Quasi-Trivial (QT) sequences in 2001 by Vannucci and Verchery [1] represented a major improvement. In [1], the authors utilised the polar formalism [3], in the framework of the Classical Laminate Theory (CLT), to derive the equations defining the general conditions for membrane-bending uncoupling and *quasi-homogeneity* (i.e. uncoupling plus equal behaviour in terms of normalised membrane and bending stiffness tensors) for a laminate made of identical plies. Indeed, QT stacking sequences are a class of exact solutions to these equations. For this reason, QT solutions have received great attention in the field of laminates design and optimisation. In [4] the authors analysed the problem of superposing laminates by means of the polar formalism and concluded that, generally speaking, the superposition of two QT stacks does not give rise to another QT one. In [5], Vannucci *et al.* proved that one can obtain fully orthotropic laminates by using QT quasi-homogeneous stacks with angle-ply orientations. These sequences were used to search optimum flexural solutions. Jibawy *et al.* [6] made use of the same idea within an optimisation procedure, in order to constrain the solutions to be quasi-homogeneous orthotropic ones. In [7], Montemurro and Catapano utilised QT quasi-homogeneous stacks in the framework of the multi-scale two-level optimisation of variable angle tow laminates.

The obstacles to a larger diffusion of QT sequences are that, firstly, only few sequences are available in the literature, while to obtain a complete database a complex combinatorial algorithm is required and no exhaustive guidelines are provided for that; secondly, such an algorithm may find QT sequences up to a limited total number of plies, due to computational limits.

The aim of this work is to contribute toward the solution of these problems. Therefore, the steps for the implementation of an efficient algorithm to find QT sequences are explained.

2 FUNDAMENTALS OF QUASI-TRIVIAL SOLUTIONS

Consider a multilayer plate composed of n plies, Figure 1. Axes x and y lay on the laminate middle plane, axis z is perpendicular to this plane. The CLT gives the constitutive relationship between generalised forces and generalised strains of the middle plane:

$$\begin{aligned} \mathbf{N} &= \mathbf{A}\boldsymbol{\epsilon}_0 + \mathbf{B}\boldsymbol{\chi}\,, \\ \mathbf{M} &= \mathbf{B}\boldsymbol{\epsilon}_0 + \mathbf{D}\boldsymbol{\chi}\,. \end{aligned} \tag{1}$$

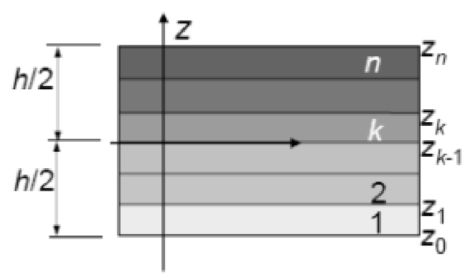

Figure 1: Laminate stack parameters and notation.

In Eq. (1), $\mathbf{N}$, $\mathbf{M}$, $\boldsymbol{\epsilon}_0$ and $\boldsymbol{\chi}$ are the vectors of in-plane resultant forces and bending moments per unit length, in-plane strains and curvatures of the middle plane of the laminate, respectively. $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{D}$ are the membrane, membrane/bending coupling and bending stiffness matrices, respectively. For a laminate with identical plies it stands:

$$\mathbf{A} = \frac{h}{n}\sum_{k=1}^{n}\mathbf{Q}(\delta_k)\,, \quad \mathbf{B} = \frac{1}{2}\frac{h^2}{n^2}\sum_{k=1}^{n} b_k\mathbf{Q}(\delta_k)\,, \quad \mathbf{D} = \frac{1}{12}\frac{h^3}{n^3}\sum_{k=1}^{n} d_k\mathbf{Q}(\delta_k)\,. \tag{2}$$

In Eq. (2) $\mathbf{Q}_k$ is the reduced stiffness matrix of the k-th ply, while δ_k is its orientation angle. Coefficients b_k and d_k depend on the position k of the ply within the stack:

$$b_k = 2k - n - 1\,, \tag{3}$$

$$d_k = 12k(k - n - 1) + 4 + 3n(n + 2)\,. \tag{4}$$

For convenience, normalised stiffness matrices are defined as follows:

$$\mathbf{A}^* = \frac{\mathbf{A}}{h}, \quad \mathbf{B}^* = 2\frac{\mathbf{B}}{h^2}, \quad \mathbf{D}^* = 12\frac{\mathbf{D}}{h^3}\,. \tag{5}$$

In addition, it is possible to define the laminate homogeneity matrix:

$$\mathbf{C} = \mathbf{A}^* - \mathbf{D}^*\,, \tag{6}$$

which measures the differences between normalised membrane and bending behaviours.

A laminate is said to be uncoupled if:

$$\mathbf{B} = \mathbf{0}\,, \tag{7}$$

while it is said homogeneous if:

$$\mathbf{C} = \mathbf{0} \tag{8}$$

Finally a laminate is *quasi-homogeneous* if properties (7) and (8) hold simultaneously.

Vannucci and Verchery [1] used the polar formalism to represent matrices $\mathbf{A}^*$, $\mathbf{B}^*$, $\mathbf{D}^*$ and $\mathbf{C}$ and to rewrite Eqs. (7) and (8) as, respectively:

$$\sum_{k=1}^{n} b_k e^{4i\delta_k} = 0, \quad \sum_{k=1}^{n} b_k e^{2i\delta_k} = 0, \tag{9}$$

$$\sum_{k=1}^{n} c_k e^{4i\delta_k} = 0, \quad \sum_{k=1}^{n} c_k e^{2i\delta_k} = 0, \tag{10}$$

where c_k is a coefficient related to matrix $\mathbf{C}$ whose expression is:

$$c_k = -2n^2 - 12k(k-n-1) - 4 - 6n\,. \tag{11}$$

We remark that coefficient b_k varies linearly with the position index k of the ply, whilst c_k is symmetric with a parabolic variation with respect to k. This is shown in Fig. 2 for the case of a laminate composed of 12 plies.

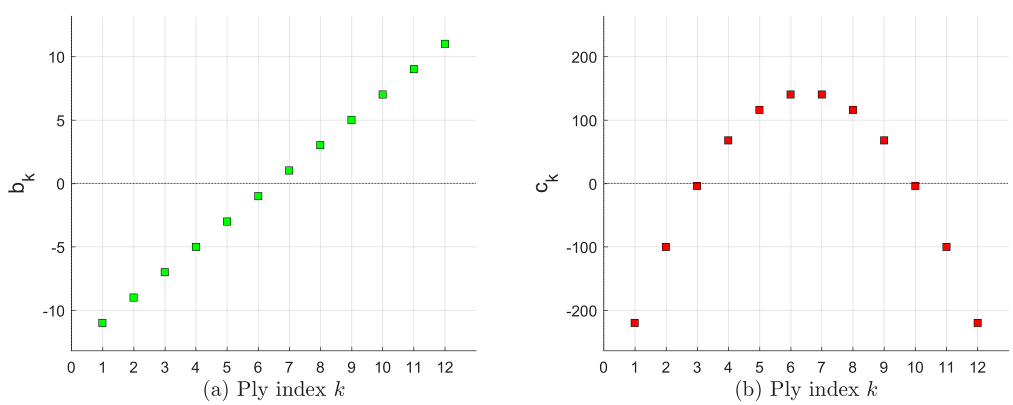

Figure 2: Trend of coefficient b_k, (a), and c_k, (b), with respect to the ply position index k.

In addition, the sum of each coefficient over the interval $[1, n]$ is always null:

$$\sum_{k=1}^{n} b_k = 0, \quad \sum_{k=1}^{n} c_k = 0. \tag{12}$$

To explain the concept of QT solutions, consider a laminate composed of n plies and m different orientation angles. Let G_j be the set of plies sharing orientation angle θ_j, i.e.

$$G_j = \{k : \delta_k = \theta_j\} \ . \tag{13}$$

The union of these sets gives the set of position indexes of the laminate, $k = 1, ..., n$.

Expressions in Eqs. (9) and (10) can be split as sums over different sets G_j, $j = 1, ..., m$:

$$\sum_{k=1}^{n} b_k e^{4i\delta_k} = \sum_{j=1}^{m} e^{4i\theta_j} \sum_{k \in G_j} b_k \ , \qquad \sum_{k=1}^{n} b_k e^{2i\delta_k} = \sum_{j=1}^{m} e^{2i\theta_j} \sum_{k \in G_j} b_k \ , \tag{14}$$

$$\sum_{k=1}^{n} c_k e^{4i\delta_k} = \sum_{j=1}^{m} e^{4i\theta_j} \sum_{k \in G_j} c_k \ , \qquad \sum_{k=1}^{n} c_k e^{2i\delta_k} = \sum_{j=1}^{m} e^{2i\theta_j} \sum_{k \in G_j} c_k \ . \tag{15}$$

It results that if the sum of coefficients b_k or c_k is null over each set G_j, then uncoupling or homogeneity requirements are satisfied, regardless the value of the orientation angle in each group. In this context, a group of plies oriented at θ_j, for which:

$$\sum_{k \in G_j} b_k = 0, \quad j = 1, ..., m \, , \tag{16}$$

$$\sum_{k \in G_j} c_k = 0 \quad j = 1, ..., m \, , \tag{17}$$

is called *saturated group* with respect to coefficients b_k or c_k, respectively; the related set of indexes G_j is called *saturated set.* A QT stack is entirely composed of saturated groups.

Since a QT stack can satisfy uncoupling, homogeneity or quasi-homogeneity conditions regardless to the value of the orientation angle characterising each saturated group, the orientation angles can be chosen/optimised to satisfy further requirements (elastic properties along some prescribed directions, buckling behaviour, natural frequencies, etc.).

3 AN EFFICIENT ALGORITHM FOR QT SOLUTION SEARCH

In [1], Vannucci and Verchery described the concepts at the basis of quasit-trivial solutions. The key is to find combinations of plies resulting in null sums of coefficient b_k and c_k for each orientation group. To this aim, an algorithm needs to be implemented. By means of an appropriate combinatorial strategy, sequences shall be generated and checked to meet Eqs. (16) and/or (17). However, in [1], no details about the implementation were given.

To this purpose, in this work a very general algorithm able to find QT solutions is outlined in Algorithm 1. This algorithm, conceptually simple, may be extremely heavy from a computational viewpoint. For example, when increasing the number of plies n of the staking sequence, the number of sequences generated in step 2 increases exponentially. This problem gives rise to two issues: the amount of memory necessary to stock all sequences increases together with the time required to check every sequence during step 3. However, by introducing the following concepts, Algorithm 1 can be modified and improved in order to reduce the computational costs. The new algorithm used to search for QT solutions is named *Improved QT stacks finder* and its logical flow is summarised in Algorithm 2.

Algorithm 1 General QT stacks finder

1. Set inputs: n and m, $n > m$;
2. Generate sequences:
   2.1 $\forall$ possible repartition of the n stack indexes among the m groups:
      2.1.1 Generate a base sequence respecting the repartition;
      2.1.2 Generate all permutations of the base sequence;
      2.1.3 Eliminate repeated sequences;
3. Search for QT solutions:
   3.1 Find sequences for which Eq. (16) is verified (uncoupled QT solutions);
   3.2 Find sequences for which Eq. (17) is verified (homogeneous QT solutions);
   3.3 Find sequences for which Eqs. (16) and (17) are verified (quasi-homogeneous QT solutions);
4. Classify QT solutions found in step 3 in two groups:
   4.1 QT dependent solutions that are discarded;
   4.2 QT independent solutions that are stored.

(the concept of dependent and independent solution will be explained later)

1. **Partial-sequences generation.** Every sequence can be considered by a composition of two sets of layers, i.e. *partial-sequences.* The layers belonging to each of these sets are associated to positive or negative values of b_k and c_k depending on their evolution with respect to k, see Fig. 2.
   Therefore, at step 2 of Algorithm 1, those sequences having an orientation group appearing only in one of the two partial-sequences are not QT solutions. Indeed, the concerned orientation group is not saturated with respect to the considered coefficient. The process can then be carried out by generating two partial-sequences, imposing that at least one ply of each group appears in both of them, and then assembling the complete sequence. In this way, a significant amount of sequences that cannot be QT solutions are neither generated nor checked, thus greatly reducing computational cost. Furthermore, the use of partial-sequences reduces memory size problems related to combinatorial calculations in the generation process. To be remarked that in some particular cases coefficients b_k and c_k may be null for a given ply position. These cases needs to be handled in the algorithm.
2. **Mechanical independence.** To understand this concept, consider sequences [1 2 3 3 2 1] and [3 2 1 1 2 3]: according to [1], these stacks are not mechanically independent because one of them can be obtained from the other one by switching one or more orientation groups. As QT solutions are exact solutions (with respect to a given criterion) regardless to the orientation values, the two sequences are indeed the same and only one has to be stored. To effectively suppress mechanically dependent solutions, the problem was faced in the generation phase: only mechanically distinct sequences are generated by the algorithm. This can be done also in the context of half-sequences generation process, by considering that particular care must be taken due to the fact that mechanical independence is to be obtained on the complete sequence and not on each half-sequence.

3. **Mathematical independence.** As an example, consider now sequences [1 2 2 2 2 1] and [1 2 3 3 2 1]: according to [1], they are not mathematically distinct, i. e. the first one can be obtained by assuming that the orientation of group 3 is equal to orientation of group 2 in the second stack. Thus, the first sequence has a saturated group (group 2) that is composed by two saturated sub-groups (groups 2 and 3 of the second sequence). In general, a sequence with a higher number of groups can be obtained from a mathematically dependent solution. On the other hand, a solution is denoted as mathematically independent when no solutions with a higher number of orientation groups can be derived from it. This leads to a major simplification in the algorithm: QT solutions with a given number of orientation groups can be obtained by mathematically dependent solution with a lower number of orientation groups. This means that the search for QT solutions with $m > 2$ may be performed directly into the *raw* (i.e. containing mathematically dependent solutions) set of solutions for the case $m = 2$. Thus, firstly QT solutions are found for the case $m = 2$. Then, the *raw* set of QT solutions is used as input for the *growing loop* (see Algorithm 2): solutions are processed and separated into three groups:
   (a) independent (i.e. both mechanically and mathematically distinct) ones, which are stored;
   (b) growable dependent ones, which will be *grown* to generate the raw set of solutions with $m = m^* + 1$ (input for the next cycle of the loop, see Algorithm 2);
   (c) dependent ones.

   To clarify the concept, consider the sequences [1 2 2 1 1 2 2 1] and [1 2 2 2 2 2 2 1]. They are not mathematically distinct from the sequence [1 2 3 4 4 3 2 1]. Therefore, in the growing loop, one of the two sequences will be discarded, while the other one will be classified as a growable one.

Tables 1, 2 and 3 report, respectively, the number of uncoupled, homogeneous and quasi-homogeneous QT solutions found in this study and in [1, 4], for comparison purposes. Firstly, Algorithm 2 is able to find QT solutions with higher number of plies, n, than in [1, 4]. Additionally, it is worth mentioning that for some given couples of values of n and m the number of uncoupled and quasi-homogeneous QT solutions found is greater than that reported in [1, 4], see Tables 1 to 3: results from [1] are reported in rows marked by the [1]-reference. For the sake of brevity, only those cases for which the number of solutions differs between the present study and [1, 4] have been reported; differences are highlighted by bold style. A detailed proof of the validity of solutions found in the present work and of the existence of more solutions with respect to those presented in [1, 4] is reported in [2]. Despite the achievement attained with Algorithm 2, computational limits to the maximum number of plies n still exist.

To overcome this limit, general rules that allow obtaining QT solutions made of a higher number of layers have been derived, exploiting the superposition of initial QT sequences, [2]. In general, the stack resulting from superposition of QT solutions is not necessarily a

	N. of groups m											
N. of plies n	2	3	4	5	6	7	8	9	10	11	12	N. of solutions
7	0	1	1	0	0	0	0	0	0	0	0	2
8	1	0	1	0	0	0	0	0	0	0	0	2
9	0	1	2	1	0	0	0	0	0	0	0	4
10	0	4	0	1	0	0	0	0	0	0	0	5
11	0	0	**9**	4	1	0	0	0	0	0	0	**14**
11-[1]	0	0	**6**	4	1	0	0	0	0	0	0	**11**
12	1	**8**	9	0	1	0	0	0	0	0	0	**19**
12-[1]	1	**4**	9	0	1	0	0	0	0	0	0	**15**
13	0	0	**25**	**32**	6	1	0	0	0	0	0	**64**
13-[1]	0	0	**14**	**20**	6	1	0	0	0	0	0	**41**
14	0	**37**	**34**	17	0	1	0	0	0	0	0	**89**
14-[1]	0	**22**	**17**	17	0	1	0	0	0	0	0	**57**
15	0	0	**10**	**207**	**78**	9	1	0	0	0	0	**305**
15-[1]	0	0	**5**	**111**	**48**	9	1	0	0	0	0	**174**
16	0	**58**	**305**	**96**	29	0	1	0	0	0	0	**489**
16-[1]	0	**29**	**168**	**48**	29	0	1	0	0	0	0	**275**
17	0	0	**2**	**893**	**895**	**144**	12	1	0	0	0	**1947**
17-[1]	0	0	**1**	**458**	**471**	**90**	12	1	0	0	0	**1033**
18	0	**114**	**1492**	**1262**	**208**	45	0	1	0	0	0	**3122**
18-[1]	0	**57**	**746**	**686**	**104**	45	0	1	0	0	0	**1639**
19	0	0	0	2216	8192	2663	264	16	1	0	0	13352
20	0	0	7391	11240	3683	396	66	0	1	0	0	22777
21	0	0	0	4936	59701	39986	6283	406	20	1	0	111333
22	0	0	29144	101207	49008	8869	694	93	0	1	0	189016
23	0	0	0	6369	346057	519231	141298	13130	626	25	1	1026737
24	0	0	75421	844224	665507	156300	18569	1118	126	0	1	1761266

Table 1: Number of independent uncoupled QT solutions obtained as a function of total number of plies and number of orientation groups

Algorithm 2 Improved QT stacks finder

1. Set inputs: n, $m = m^* = 2$;
2. Generate sequences:
   - perform partial-sequences generation;
   - generate only mechanically distinct complete sequences;
3. Search for QT solutions:
   3.1 Find sequences for which Eq. (16) is verified (uncoupled QT solutions);
   3.2 Find sequences for which Eq. (17) is verified (homogeneous QT solutions);
   3.3 Find sequences for which Eqs. (16) and (17) are verified (quasi-homogeneous QT solutions);
4. *Growing* loop of growable dependent solutions:
   4.1 Input: raw set of QT solutions with $m = m^*$;
   4.2 Classify QT solutions in three groups:
      4.2.1 Independent QT solutions with $m = m^*$, that are stored;
      4.2.2 Dependent QT solutions, that are discarded;
      4.2.3 Growable dependent QT solutions, that are grown to $m = m^* + 1$ and temporarily stored;
   4.3 Repeat Step 4 until the set of growable dependent solution is empty.

	N. of groups m					
N. of plies n	2	3	4	5	6	N. of solutions
4	2	0	0	0	0	2
5	2	0	0	0	0	**2**
5-[4]						**1**
6	4	0	0	0	0	4
7	0	1	0	0	0	1
8	8	0	0	0	0	8
9	4	0	0	0	0	**4**
9-[4]						**2**
10	20	8	0	0	0	**28**
10-[4]						**22**
11	0	22	0	0	0	**22**
11-[4]						**14**
12	36	0	0	0	0	**36**
12-[4]						**34**
13	16	52	0	0	0	**68**
13-[4]						**36**
14	2	12	32	128	16	**190**
14-[4]						**119**
15	0	100	0	0 0	0	**100**
15-[4]						**52**
16	0	32	40	16	32	**120**
16-[4]						**76**
17	142	652	32	0	0	**826**
17-[4]						**445**
18	34	720	336	16	0	**1106**
18-[4]						**617**
19	4	1436	4232	512	0	6184
20	68	4856	5104	0	0	10028
21	26	500	1168	1248	0	2942
22	0	36804	302832	139424	4864	483924
23	50	164918	129212	2016	0	296196
24	152	5864	159632	0	0	165648
25	0	314018	665512	123044	4000	1106574

Table 2: Number of independent QT solutions with $\mathbf{C} = \mathbf{0}$ obtained as a function of total number of plies and number of orientation groups

QT one [4], therefore, criteria must be derived to ensure that resulting stacks are still QT ones. For more details on the derivation of such criteria the reader is addressed to [2].

	N. of groups m					
N. of plies n	2	3	4	5	6	N. of solutions
7	1(1)	0	0	0	0	1(1)
8	1	0	0	0	0	1
9	0	0	0	0	0	0
10	0	0	0	0	0	0
11	**4(2)**	0	0	0	0	**4(2)**
11-[1]	**3(2)**	0	0	0	0	**3(2)**
12	1	0	0	0	0	1
13	**4**	**3**	0	0	0	**7**
13-[1]	**2**	**2**	0	0	0	**4**
14	0	2(1)	0	0	0	2(1)
15	**4**	**3**	0	0	0	**7**
15-[1]	**2**	**2**	0	0	0	**4**
16	**6**	3(1)	0	0	0	**9(1)**
16-[1]	**5**	3(1)	0	0	0	**8(1)**
17	**30**	**11**	0	0	0	**41**
17-[1]	**15**	**8**	0	0	0	**23**
18	0	**9**	0	0	0	**9**
18-[1]	0	**5**	0	0	0	**5**
19	**60**	**41**	0	0	0	**101**
19-[1]	**30**	**22**	0	0	0	**52**
20	**52**	**17**	1	0	0	**70**
20-[1]	**30**	**9**	1	0	0	**40**
21	**62**	**18(2)**	0	0	0	**80(2)**
21-[1]	**31**	**13(2)**	0	0	0	**44(2)**
22	**32(2)**	**188(1)**	**26**	2	0	248(3)
22-[1]	**17(2)**	**98(1)**	**13**	2	0	**130(3)**
23	**189(1)**	**970**	0	0	0	**1159(1)**
23-[1]	**95(1)**	**499**	0	0	0	**594(1)**
24	**248**	**47**	1	0	0	**296**
24-[1]	**140**	**26**	1	0	0	**167**
25	**326**	**4184**	**98**	0	0	**4608**
25-[1]	**163**	**2132**	**57**	0	0	**2352**
26	**108**	**2065 (2)**	**672 (3)**	**41 (2)**	**3**	**2889 (7)**
26-[1]	**54**	**1059 (2)**	**354 (3)**	**26 (2)**	**2**	**1495 (7)**
27	**171(1)**	**1804**	**510**	**39**	1	**2525(1)**
27-[1]	**86(1)**	**918**	**256**	**21**	1	**1282(1)**
28	**357**	**9492(1)**	**1691(2)**	**61**	**9**	**11610(3)**
28-[1]	**203**	**4789(1)**	**871(2)**	**33**	**6**	**5902(3)**
29	**122**	**75281**	**15068**	**167**	0	**90638**
29-[1]	**61**	**37747**	**7546**	**86**	0	**45441**
30	**106**	**10923**	**1009(3)**	**51**	0	**12089(3)**
30-[1]	**53**	**5552**	**512(3)**	**29**	0	**6146(3)**
31	28	290227	156565(1)	1728	1	448549(1)
32	263	161436(5)	70091	4521	100	236411(5)
33	316	260442	112324	937	0	374019
34	716	1389039 (107)	568492 (35)	12589	38	1970874 (142)
35	2	8291650 (8)	6392064 (7)	90433	82	14774231 (15)

Table 3: Number of independent quasi homogeneous QT solutions obtained as a function of total number of plies and number of orientation groups; symmetric solutions are reported in parentheses.

4 CONCLUSIONS

In this paper, a detailed description of an efficient algorithm for the search of QT sequences has been presented. The algorithm has proved to be able to find longer sequences than in the past. In addition, a greater number of solutions has been found, showing that previous solutions number was underestimated.

In addition, superposition criteria derived in [2] allow obtaining QT sequences by superposing basic QT thinner stacks by following precise rules. In this way, QT solutions with any desired total number of plies can be generated. This virtually shatters any obstacle to the adoption of QT sequences for the design of thick laminates and laminates made of a significant amount of *thin plies*.

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REFERENCES

[1] P. Vannucci, G. Verchery, A special class of uncoupled and quasi-homogeneous laminates, Composites Science and Technology 61 (10) (2001) 1465 – 1473.

[2] T. Garulli, A. Catapano, M. Montemurro, J. Jumel, D. Fanteria, Quasi-trivial stacking sequences for the design of thick laminates, Composites Structures (submitted).

[3] G. Verchery, Les Invariants des Tenseurs d'Ordre 4 du Type de l'Élasticité, Springer Netherlands, Dordrecht, 1982, pp. 93–104.

[4] P. Vannucci, G. Verchery, Stiffness design of laminates using the polar method, International Journal of Solids and Structures 38 (50) (2001) 9281 – 9294.

[5] P. Vannucci, R. Barsotti, S. Bennati, Exact optimal flexural design of laminates, Composite Structures 90 (3) (2009) 337 – 345.

[6] A. Jibawy, B. Desmorat, A. Vincenti, Structural rigidity optimization of thin laminated shells, Composite Structures 95 (2013) 35 – 43.

[7] M. Montemurro, A. Catapano, On the effective integration of manufacturability constraints within the multi-scale methodology for designing variable angle-tow laminates, Composite Structures 161 (2017) 145 – 159.