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SCALABLE ALGORITHMS FOR PDE-CONSTRAINED OPTIMIZATION UNDER UNCERTAINTY

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We consider optimization problems governed by PDEs with infinite dimensional random parameter fields. Such problems arise in numerous applications: optimal design/control of systems with stochastic forcing or uncertain material properties or geometry; inverse problems governed by stochastic forward problems; or Bayesian optimal experimental design problems with the goal of minimizing the uncertainty or maximizing the information gain in the inferred parameters.

Standard Monte Carlo evaluation of the objective results in a number of PDE constraints equal to the number of samples. The resulting many-PDE-constrained optimization problem, while deterministic, is nevertheless prohibitive to solve, especially when the PDEs are "complex" (large-scale or nonlinear or coupled) and when the parameter space is high-dimensional.

We present high-order derivative-based approximations of the parameter-to-objective maps that, in combination with randomized algorithms, exploit the structure of these maps (smoothness, low effective dimensionality). Their use as a basis for variance reduction is demonstrated to significantly accelerate Monte Carlo sampling and lead to scalable solution of several stochastic optimization problems governed by turbulent flow and wave scattering with up to $O(10^6)$ uncertain parameters.

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