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Integration of data in finite element methods for computational mechanics

E. Burman[1], D. Elfversson[2], P. Hansbo[3], M. Larson[2], K. Larsson[2] and L. Oksanen[1]

[1] Department of Mathematics, University College London, Gower Street, London UK, WC1E 6BT, e.burman@ucl.ac.uk, l.oksanen@ucl.ac.uk

[2] Department of Mathematics and Mathematical Statistics, Umeå University, SE–901 87 Umeå, Sweden, daniel.elfverson@umu.se, karl.larsson@umu.se, mats.larson@umu.se

[3] Department of Mechanical Engineering, Jönköping University, SE-551 11 Jönköping, Sweden, peter.hansbo@ju.se

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The two necessary ingredients in a model for computational mechanics are the mechanical model in the form of a partial differential equation and data. The latter can be measurments of the solution itself or of design parameters, such as the computational geometry. Traditionally most research efforts have been oriented towards the efficient and robust solution of the partial differential equation, assuming that accurate data is available on a form the makes the physical model mathematically well-posed. The treatment of data and integration of data in computation has typically been treated separately. In the case of geometry data through mesh generation and in the case of assimilation of measured data through the theory of ill-posed problems with regularization on the continuous level and then discretization. Many computational problems, such as multiphysics problems or shape optimization problems, however require not only seamless integration of geometric data but also flexibility in changing the geometry through the computation. Data assimilation using Tikhonov regularization on the other hand leads to a perturbed physical model and optimal convergence of numerical approximations is typically prohibited by the consistency error.

In this talk we will report on some new approaches for the integration of data with computation, focus will be on methods that are well balanced so that they remain robust without sacrificing accuracy. For the integation of geometry data, we will consider recent developments of cut finite element methods [2], that is, finite element methods that allow for meshes that do not fit the computational geometry, but still remain accurate and robust. Such methods are particularly well suited for data given implicitly by a level set or indicator function that can be transformed throughout the computation, thus changing the geometry. In this framework we will discuss some theoretical results showing that the performance of the method is similar to that of a standard finite element method on meshed domains. We will then show some examples of how the generality of the framework allows for the computation of complex phenomena such as flow in fractured media, fluid-structure interaction with contact or shape optimization [3].

In the second part of the talk we will focus on the integration of measured data in computation. Here typically the problems are ill-posed and we will present a framework for weakly consistent regularization, on the discrete level, for data assimilation problems [1]. Tools from stabilized finite element methods are reintroduced in this new context, serving as regularization terms. We then discuss how the consistency and numerical stability of the methods can be used in combination with the conditional stability of the (continuous) data assimilation problem, quantified in the form of Carleman estimates, to prove optimal error estimates. The discussion will be illustrated by some model examples: the elliptic Cauchy problem, computational unique continuation for Helmholtz problem and the data reconstruction problem for the heat equation [4].

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