

Numerical analysis of a class of staggered finite volume schemes for Lagrangian Hydrodynamics

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Keywords: *Finite Volume scheme, entropy conservative scheme, gas dynamics, staggered Lagrangian hydrodynamics*

Following the pioneer work of von Neumann and Richtmyer in [1], staggered grid hydrodynamics (SGH) , is a class of numerical discretizations based on staggered and moving meshes, designed to solve multidimensional Lagrangian gas dynamic equations. In SGH, all the thermodynamic variables of the fluid, such as density, pressure and internal energy are cell-centered, whereas the velocity is defined at the nodes of the mesh. The main requirements for such numerical methods are to ensure the local mass, momentum and total energy conservations.

These points are addressed by a particular structure of the discrete div and grad operators, as well as the discretization of the momentum equation on dual lagrangian cells centered around the nodes of the primal mesh, and the discretization of the internal energy equation. The procedure to derive such schemes enters the framework of mimetic methods [2, 3]. Finally, the dissipation of kinetic energy into internal energy through shock waves is ensured by means of an artificial viscosity.

We will propose a numerical analysis of those schemes, adopting a similar approach to the one developed in [4] for a class of staggered finite volume schemes for Eulerian hydrodynamics. We will present the main properties such as Lax consistency, weak entropy inequality, or the stability of the solution, including some relevant numerical results to illustrate.

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