

# P-MULTIGRID PRECONDITIONERS APPLIED TO HIGH-ORDER DG AND HDG DISCRETIZATIONS

Matteo Franciolini<sup>1,2</sup>, Krzysztof Fidkowski<sup>1</sup> and Andrea Crivellini<sup>2</sup>

<sup>1</sup> Department of Aerospace Engineering, University of Michigan, Ann Arbor, USA

<sup>2</sup> DIISM, Università Politecnica delle Marche, Ancona, Italy

**Key words:** *high-order, DG, HDG, p-Multigrid, preconditioners, parallel efficiency*

In recent years, high-order discontinuous Galerkin methods have become increasingly popular in the field of Computational Fluid Dynamics. This fact is certainly ascribed to their convenient dispersion and diffusion properties, the ease of parallelization thanks to their compact stencil, and their accuracy in arbitrary complex geometries. However, the implementation of an efficient solution strategy is still subject of active research, especially for unsteady flow problems [1] involving the solution of the Navier–Stokes equations. In this work the use of a p-multigrid preconditioned flexible GMRES solver to deal with the solution of stiff linear systems arising from high order time discretization is explored in the context of two high-order spatial discretizations. The first one is a standard modal discontinuous Galerkin (DG) solver [2], while the second one is an hybridizable discontinuous Galerkin (HDG) [3] discretization, which for high order has fewer globally-coupled degrees of freedom compared to DG. The effects of several parameters involving the use of the p-multigrid preconditioner, such as the choice of the smoother type and the number of smoothing iterations, as well as the scalability of the algorithms, are considered and compared to standard single-grid preconditioners like block/line–Jacobi or ILU(0). The efficiency of the solution strategies is assessed using different time discretization schemes, that will be applied to problems of growing complexity. Preliminary results on two-dimensional cases show that, while for both the space discretizations a considerable reduction in the number of GMRES iterations is achieved, only within the DG discretization is this gain reflected on the CPU time. In view of the numerical results, the convenience of using the former or the latter space discretization will be discussed.

## REFERENCES

- [1] Franciolini, M., Crivellini, A. and Nigro, A. On the efficiency of a matrix-free linearly implicit time integration strategy for high order discontinuous Galerkin solution of incompressible turbulent flows. *Comput. Fluids* (2017) **159**, pp. 276–294.
- [2] Fidkowski, K. J. Output-Based Error Estimation and Mesh Adaptation for Steady and Unsteady Flow Problems. *38<sup>th</sup> Advanced VKI CFD Lecture Series* (2015), Von Karman Institute for Fluid Dynamics.
- [3] Fidkowski, K. J. A hybridized discontinuous Galerkin method on mapped deforming domains. *Comput. Fluids* (2016) **139**, pp. 80–91.