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Multiscale Hybrid-Mixed Methods for the Stokes and Brinkman Equations

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This work presents a family of finite element methods for multiscale fluid problems, named Multiscale Hybrid-Mixed (MHM) methods. The MHM method is a consequence of a hybridization procedure which characterizes the unknowns as a direct sum of a "coarse" solution and the solutions to problems with Neumann boundary conditions driven by the multipliers. As a result, the MHM method becomes a strategy that naturally incorporates multiple scales through multiscale basis functions while providing solutions with high-order precision for the primal and dual variables. The completely independent local problems are embedded in the upscaling procedure, and then, computational approximations may be naturally obtained in a parallel computing environment. The numerical analysis for the one- and two-level versions of the MHM method shows that the method is optimal convergent and achieves super-convergence for the locally conservative velocity field. A face-based a posteriori estimator is proposed which is locally efficient and reliable with respect to the natural norms. The general framework and some recent results are illustrated for the Stokes and Brinkman equations and validated through a large variety of numerical results for highly heterogeneous coefficient problems.

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