

# AN EXPLORATION INTO THE PERFORMANCE OF A NOVEL FINITE ELEMENT ALGORITHM FOR SOLUTION OF THE BOLTZMANN EQUATION

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Knudson Number, Boltzmann Equation, CFD, Computational Methods, Kinetic Theory, Riemann problem, Reverse Magnus

## Abstract.

A Boltzmann Solver algorithm has been developed via a the Bhatnagar–Gross–Krook (BGK) operator to express the Right Hand Side Collision Term of the Boltzmann Equation. The algorithm is based on a two-step discontinuous Taylor Galerkin scheme that utilises a discontinuous finite element discretisation for the physical space representing the geometry and a high order discretisation for molecular velocity space describing the molecular distribution function.

The algorithm developed has key advantages over continuum methods specifically in problems where discontinuities prevail such as hypersonic flow where features like shock waves are quite common and rarefied flow where molecular level effects are no longer negligible. The Knudson Number ( $Kn$ ) has been used to identify and classify various types of flow to be modelled by the algorithm. A space re-entry to the atmosphere has been modelled to demonstrate the capabilities of the solver and how varying initial parameters would affect the output. Also, the Riemann problem has been tackled to explore the capabilities of the algorithm into being adopted for high-vacuum problem and initial discontinuities. Tackling the Riemann Problem can unlock the potential for further explorations into molecular flow at nano levels to demonstrate the capability to capturing micro properties and their variations.

This work shows that the tested algorithm is capable of solving flow problems across a range of length scales hence validates the potential capabilities of the algorithm and its unique capability of being utilized across diverse range of flow problems.

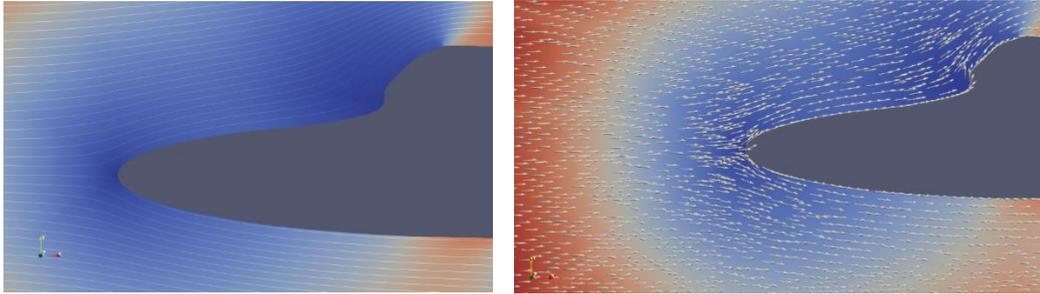


Figure: Left visualization shows the density contours around the space re-entry problem and the right visualization shows the velocity magnitude contours and vector directions. This shows the algorithm's capability at Knudsen Number in excess of 100 and Mach 5.0 + (hypersonic flight)

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