

A time adaptive multirate Neumann-Neumann waveform relaxation method for thermal fluid-structure interaction

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The efficient simulation of thermal interaction between fluids and structures is crucial in the design of many industrial products, e.g. thermal anti-icing systems of airplanes, gas quenching, which is an industrial heat treatment of metal workpieces or the cooling of rocket thrust chambers.

Unsteady thermal fluid structure interaction is modelled using two partial differential equations describing a fluid and a structure which are coupled at an interface. The standard algorithm to find solutions of the coupled problem is the Dirichlet-Neumann iteration, where the PDEs are solved separately using Dirichlet-, respectively Neumann boundary with data given from the solution of the other problem. Previous analysis and numerical experiments show that this iteration is fast for the thermal coupling of air and steel [2]. This method has two main disadvantages: the subsolvers are sequential and both fields are solved with a common time resolution. Using instead a time adaptive multirate scheme would be more efficient.

In view of this, we present here a high order, parallel, time adaptive, multirate numerical method for two heterogeneous coupled heat equations. We use the Neumann-Neumann waveform relaxation (NNWR) method which is a variant of WR methods based on the classical Neumann-Neumann iteration [1]. When choosing the relaxation parameter right, one iteration is sufficient. We present an analysis of the NNWR algorithm that shows that the optimal relaxation parameter is highly dependent on the material coefficients. In order to get an adaptive multirate scheme, we use possibly different adaptive temporal discretization methods on the two subdomains. Furthermore, two time integration alternatives are analyzed, the implicit Euler method and a second order singly diagonally implicit Runge-Kutta method (SDIRK2). Numerical results show that second order is achieved even when using linear interpolation in the multirate case.

REFERENCES

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