

Development of methods for weak imposition of Dirichlet boundary conditions for fictitious finite element analysis

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Keywords: *Dirichlet boundary condition, fictitious domain*

One of the major difficulties with the fictitious domain finite element analysis methods comes from incorporating Dirichlet boundary conditions (DBC). Unlike the classical finite element method, where DBCs can be imposed directly on nodes, weak imposition methods need to be considered. In this presentation the weak forms, energy functionals and formulations of different types of approaches developed so far are discussed and compared.

The IB method is considered in more detail. We follow the approaches of Kumar et al.[1], in which step-like IB functions are employed to weight the unknowns of the background grid, leading to the final system of equations in which the DBC imposition can be categorized as a penalty formulation. Here we provide an alternative derivation of the IB method for solid mechanics problems that is capable of simulating inclined boundaries that are not fully fixed, such as roller boundaries not parallel to one of the coordinate axes. Compared to previous approaches, a simpler system of equations with only one additional term in the stiffness matrix is obtained. The LLM method is a recently developed two-field approach that in solid mechanics problems uses the stress near the boundary as the Lagrange multiplier to weakly impose DBCs. Following the symmetric version of LLM method in [2], again we modify the original formulation such that the method can be applied for roller boundary conditions, which is helpful in engineering applications.

These two methods are then extended to the non-linear cases of frictional boundaries and elasto-plasticity. The implementation techniques of the non-linear problems are discussed, and convergence rates of uniform refinements similar to classical finite element analysis are achieved.

REFERENCES

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