

MPDATA: Third-order accuracy for variable flows

Maciej Waruszewski^{1*}, Christian Kühnlein², Hanna Pawlowska¹
and Piotr K. Smolarkiewicz²

¹ Institute of Geophysics, Faculty of Physics, University of Warsaw
Warsaw, Poland

{mwarusz, hanna.pawlowska}@igf.fuw.edu.pl

² European Centre for Medium-Range Weather Forecasts
Reading, RG2 9AX, United Kingdom

{piotr.smolarkiewicz, christian.kuehnlein}@ecmwf.int

Keywords: *Multidimensional advection, High-order schemes, Nonoscillatory schemes, Implicit large-eddy simulations, Spurious vortices*

We extend the structured-grid multidimensional positive definite advection transport algorithm (MPDATA) [1] to third-order accuracy for temporally and spatially varying flows. This is accomplished by identifying the leading truncation error of the standard second-order MPDATA, performing the Cauchy-Kowalevski procedure to express it in a spatial form and compensating its discrete representation—much in the same way as the standard MPDATA corrects the first-order accurate upwind scheme. The procedure of deriving the spatial form of the truncation error was automated using a computer algebra system, enabling various options in MPDATA to be included straightforwardly in the third-order scheme. Following the spirit of MPDATA, the error is compensated using the upwind scheme resulting in a sign-preserving algorithm, and the entire scheme can be formulated using only two upwind passes. Established MPDATA enhancements, such as formulation in generalised curvilinear coordinates, the nonoscillatory option or the infinite-gauge variant, carry over to the fully third-order accurate scheme.

Global tracer-transport benchmarks demonstrate benefits for chemistry-transport models fundamental to air quality monitoring, forecasting and control. A series of explicitly-inviscid implicit large-eddy simulations of a convective boundary layer [2] and explicitly-viscid simulations of a double shear layer [3] illustrate advantages of the fully third-order-accurate MPDATA for fluid dynamics applications.

REFERENCES

- [1] L. Margolin, P. K. Smolarkiewicz, Antidiffusive velocities for multipass donor cell advection, *SIAM J. Sci. Comput.*, Vol. **20**, pp. 907–929, 1998.
- [2] L. G. Margolin, P. K. Smolarkiewicz, Z. Sorbjan, Large-eddy simulations of convective boundary layers using nonoscillatory differencing *Physica D*, Vol. **133**, pp. 390–397, 1999.
- [3] D. L. Brown, M. L. Minion, Performance of under-resolved two-dimensional incompressible flows simulations, *J. Comput. Phys.*, Vol. **122**, pp. 165–183, 1995.