Stabilization of the transpose convection term in incompressible Navier-Stokes adjoint equations

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ABSTRACT

The adjoint method has been an active field of research in CFD over the last 15 years or so, thanks to the seminal works of Lions ([15]), Pironneau & al ([1], [16]), Jameson ([2], [17]), Giles and Pierce ([3]), and Loehner and Soto ([4]), just to name a few. It indeed has been recognized as the tool of choice for gradient-based optimization of internal and external flows and its applicability in an industrial context has been widely acknowledged (see [5]) for various applications, such as power losses minimization of ducted flows, or aerodynamic shape design improvement of car vehicles.

Although there have been significant developments and sophistications of the method since its early introduction, including derivation of adjoint turbulence models ([6], implementation of transient adjoints ([7]), efforts towards fully automated CAD-based ([8], [9]) or CAD-free ([8]) shape optimization methodologies, not to mention the various discussions about compared merits of discrete and continuous approaches ([10], [11]), one long standing fundamental issue has not received yet a definitive answer and remains: stability of the steady-state incompressible Navier-Stokes adjoint equations solvers for industrial applications.

The so-called Adjoint Transpose Convection (ATC) has long been identified as the troublemaker and the main source of instability ([5]). Various strategies have been proposed by several authors to alleviate the issue, including the derivation of an alternative expression that does no involve any derivative of the adjoint velocity, together with a selective damping methodology ([12]), the resort to block-coupled solvers ([13]) and specific under-relaxation of the targeted term ([14]). To the best of the authors knowledge, however, even if they do help, none of these strategies has proven to be fully satisfactory.

The present work aims at describing the mechanism itself behind the instability and its connection to the primal solver convergence behavior. In particular, it shall be emphasized how it is related to the linear stability of full Newton iterations of the primal solver and the presence of potential limit cycles. Also, it will be argued that, contrarily to a common belief ([5]), the explicit treatment of the ATC is not necessarily the main root of instability. Indeed, although it is true that the term introduces a high degree of coupling between the three components of the velocity, some theoretical analysis and numerical evidence tend to outline the role played by the implicit inertial relaxation (aka local time-stepping) of the momentum equations.

A comparison of the standard 1st order (involving adjoint velocity derivative) and the popular alternative 0th order (not involving adjoint velocity derivative) formulations of the ATC will be carried out, and a third form shall be proposed, that involves the introduction of a specific conservative flux.

Finally, a new selective under-relaxation strategy shall be presented and its effectiveness in stabilizing industrial adjoint simulations, demonstrated.

keywords: *adjoint transpose convection, incompressible Navier-Stokes, adjoint equations, under-relaxation, solver stability*

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