

# Theoretical and Computational Studies of a Rectangular Finite Wing Oscillating in Pitch and Heave

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This paper will analyse the extent of validity of Unsteady Lifting-Line Theory (ULLT) for finite wings undergoing pitch-heave oscillations in the low Reynolds number regime using Computational Fluid Dynamics (CFD).

The study of oscillating finite wings is relevant in applications such as flexible aircraft and micro air vehicles with flapping wings that aim to emulate insect flight. In both cases, finite wings undergo pitch and heave oscillations either due to aeroelasticity or excitation.

During oscillation, the wing's bound vortex oscillates with respect to time. To satisfy Kelvin's condition, vorticity is convected into the wake resulting in a periodic vorticity distribution. Helmholtz's first theorem requires that wings bound vorticity must also be shed into the wake. Hence the vorticity wake varies in both streamwise and spanwise directions. The wake influences the flow incident on the wing.

During small amplitude oscillation, the flow over the wing is mostly attached and vorticity is shed from the trailing edge into the wake. The wake can be reasonably approximated as a planar vorticity distribution. Large amplitude oscillations introduce additional phenomena to the flow. The wake becomes significantly non-planar. Flow separation may occur on the wing surface. When separation progresses to or erupts at the leading edge, leading edge vortices are shed.

Sclavounos' ULLT [2] assumes a finite planar unswept wing undergoing small sinusoidal pitch or heave oscillations in an invicid, incompressible fluid. It is comprised of an outer solution and an inner solution. The outer solution acts on the length scale of the wing span, ignoring the comparatively small chord-scale detail. The wing is considered as a sinusoidally oscillating bound vorticity concentrated on the semichord. The wake is modelled as a planar vorticity distribution stretching from the semichord infinitely far downstream. This vorticity distribution is periodic in the streamwise direction due to the oscillatory bound vortex. Additionally it varies in the spanwise direction, due to the finite nature of the wing. The wake leads to a downwash on the wing affecting the inner solution. The inner solution considers the chord at a single position in the spanwise direction. The chord is modelled as a flat plate undergoing sufficiently small oscillations to be approximated as an oscillatory velocity boundary condition on the plane. The

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resulting forces are similar to those from Theodorsen's formulation. By approximating the inner solution as a oscillatory point vortex and the corresponding planar wake, it can be matched with the outer solution. Matching allows the downwash from the three dimensional wake of the outer solution to impact the inner solution.

The primary limitations of Sclavounos' ULLT are the assumption of small displacement and attached flow. Additionally, the outer solution assumes that the wing span length scale is large compared to the chord length scale. This limits the validity of the method for small aspect ratio wings.

In an investigation into the limits of validity of Theodorsen's theory, McGowan et al.[1] demonstrated the surprising resilience of this theory in predicting pitch-heave lift cancellation, even when some of the underlying assumptions are violated. Given the similarity between the Sclavounos inner solution and Theodorsen's theory, it is hoped that an analogous behavior can be found for Sclavounos' theory.

This paper will apply Sclavounos' ULLT to rectangular wings of various aspect ratios oscillating in pitch and heave at a variety of chord reduced frequencies, span reduced frequencies and amplitudes. The consequences of clearly violating the planar wake and attached flow assumptions will be explored. The results will be compared to that predicted by Theodorsen based strip theory, high-fidelity CFD data and to preexisting experimental data where possible. A pitch-heave lift cancellation case based on Sclavounos' theory will also be examined for small and large amplitude oscillations.

## REFERENCES

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