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A SEMI-INTRUSIVE MINIMISING RESIDUAL METHOD FOR UNCERTAINTY PROPAGATION IN FLUID DYNAMICS

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A novel approach for the forward propagation of parametric uncertainties in partial differential equations (PDEs) is proposed. Classically, non-intrusive uncertainty quantification (UQ) is based on sampling from a black-box solver and used to construct a surrogate model in the parametric space, from which one can extract stochastic properties of the uncertain solution. Existing methods solely use the black-box solver as a tool to sample from the model. Fully intrusive UQ methods require the underlying model equations to be altered, which removes the possibility of using highly optimised black-box solvers.

However, when the underlying model is given in terms of a PDE, then information from the residual of this PDE can be used to effectively refine the surrogate model, by placing extra samples in the parametric space where the residual is high.

$$z_{new} = \max_{z \in I_z} | \, L(\tilde{u}(z)) - f \, | \, ,$$

Where L is the PDE operator, $\tilde{u}$ the surrogate, f the right-hand-side of the PDE and z the uncertain parameters. This procedure requires only a slight alteration of the black-box, such that it not only returns the solution value, but also the partial derivatives of the solution with respect to the spatial/temporal-coordinates, as present in the PDE. These derivatives are used to compute the PDE residual when using the surrogate as a solution. This results in a method, which tries to minimise the residual and therefore the error with respect to the exact solution in parametric space. The magnitude of the residual is not only useful to adaptively sample the model, but it also serves as a reliable error measure. The fact that we use knowledge of the underlying PDE results in a method, which is more efficient than classical non-intrusive approaches, such as stochastic collocation. The proposed method is demonstrated by applying it to several fluid dynamics problems.