

# MATHEMATICAL AND NUMERICAL MODELLING OF ACOUSTIC WELL STIMULATION

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This work presents a mathematical model and a numerical method to simulate an acoustic well stimulation (AWS) procedure for enhancing the permeability of the rock formation surrounding oil and gas wells. The AWS procedure considered herein aims to exploit the well-known permeability-enhancing effect of mechanical vibrations in acoustically porous materials, by transmitting time-harmonic acoustic waves from a sound source device, placed inside the well, to the well perforations made into the formation. The efficiency of the AWS is assessed by quantifying the amount of acoustic energy transmitted from the source device to the rock formation in terms of the emission frequency and the well configuration. A simple methodology to find optimal emission frequencies for a given well configuration is presented. The proposed model is based on the Helmholtz equation and an impedance boundary condition that effectively accounts for the porous solid-fluid interaction at the interface between the rock formation and the well perforations. Exact non-reflecting boundary conditions derived from Dirichlet-to-Neumann maps are used to truncate the circular cylindrical waveguides considered in the mathematical model. The resulting boundary-value problem is then numerically solved by means of the finite element method. Numerical examples are presented in order to demonstrate the effectiveness of the proposed method for finding optimal emission frequencies.

## REFERENCES

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