6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

Four Types of Dependencies within the Fuzzy Analysis

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Keywords: *Uncertain structural analysis, Uncertainty quantification, Fuzzy analysis*

Uncertain structural analysis is the computation of uncertain structural response for uncertain input quantities, based on a computational model ξ. If only rare data is available, the usage of the uncertainty model fuzziness is common. The computation of fuzzy responses with respect to predefined fuzzy inputs is called fuzzy analysis

$$\xi^{\mathrm{f}}\colon \mathcal{F}(\mathbb{R},[0,1]) \to \mathcal{F}(\mathbb{R},[0,1])\colon \underbrace{\underline{x}^{\mathrm{f}}}_{\text{prior dependency}} \overbrace{(\tau,\underline{\theta})}^{\text{functional dependency}} \mapsto \underbrace{\underline{z}^{\mathrm{f}}}_{\text{posterior dependency}} \overbrace{(\tau,\underline{\theta})}^{\text{functional dependency}} . \tag{1}$$

In terms of structural analysis, four kinds of dependencies are identified, as can be seen in Eq. (1).

Firstly, the input parameters $\underline{x}^{\mathrm{f}}$, i.e. material characteristics or loading conditions, may vary with time τ and/or space $\underline{\theta}$. This feature is called functional input dependency $\underline{x}(\tau,\underline{\theta})$ and could be described by fuzzy processes or fuzzy fields.

Secondly, the set of input parameters $\underline{x}^{\mathrm{f}}$ can be internally dependent, which means not all combinations of parameter values are permissible with regard to the computational model. This pre-condition is the prior dependency, which mainly depends on the definition of the multi-dimensional membership function $\mu(\underline{x})$. For random variables, the term correlation is established in this context.

Thirdly, the result quantities (e.g. stresses, strains, damages, ...) are time and spatial dependent as well. This functional output dependency $\underline{z}(\tau,\underline{\theta})$ is always given, if the finite element method is used as fundamental solution and the results are not reduced to a small amount of Quantities of Interest.

Fourthly, the result parameters $\underline{z}^{\mathrm{f}}$ can be internally dependent, called posterior dependency. This fact is commonly ignored, because fuzzy result quantities are computed for separated deterministic results. For instance, stresses $\mu(z_1 = \sigma)$ and strains $\mu(z_2 = \varepsilon)$ (at one point in space and same time) are computed independently, which subsequently yields a significant overestimation of uncertainty, since for most of undamaged elastic solid materials high stresses come alone with high strains. The challenge is to compute the membership function which depends on a vector of result quantities $\mu(\underline{z})$.

The goal of this contribution is to discuss the different kinds of dependencies and present possible solution strategies.