

Towards optimal hp approximation spaces for high-order methods: A continuous mesh approach

Georg May¹, Ajay Rangarajan² and Ankit Chakraborty³

¹ Professor, RWTH Aachen, may@aices.rwth-aachen.de

² Doctoral Candidate, RWTH Aachen, ajay.rangarajan@rwth-aachen.de

³ Graduate Student, RWTH Aachen, ankit.chakraborty@rwth-aachen.de

Keywords: *hp-adaptation, continuous mesh, discontinuous Galerkin methods.*

Adaptive meshing is widely recognized as an important tool in the numerical solution of PDE problems. This is true, in particular, for convection-diffusion problems. Numerical schemes using piecewise polynomial approximation offer increased flexibility by adjusting the local polynomial degree as well (hp -adaptation). In this context, metric-based methods have emerged as an interesting paradigm. A metric-conforming mesh is a triangulation whose elements are (nearly) equilateral under the Riemannian metric induced by a tensor-valued mapping, the continuous mesh. Manipulating the metric can be done by analytical optimization methods, followed by mesh re-generation using a standard metric-based mesh generator.

Previously, we have proposed a continuous-mesh adaptation method for higher order discontinuous Galerkin methods on triangular meshes [1]. A metric is obtained by a two-step formal optimization procedure with respect to a recently proposed error model [2]. More recently, we have begun to investigate extensions to goal-oriented adaptation, and/or hp -adaptation. At the same time an extension to 3D and tetrahedral grids is currently under development. These extensions are the main focus of this talk. Challenges include the derivation of suitable continuous-mesh error models. In the case of goal-oriented adaptation, these usually incorporate the solution of an adjoint problem. Furthermore, methods are needed which determine the optimal continuous mesh with respect to the error models. These optimization techniques are complicated by the presence of variable degree of polynomial approximation (hp -adaptation). Furthermore, the optimization of the three-dimensional metric, while conceptually similar to the two-dimensional case, is considerably more involved.

In addition to the theory, we present several numerical examples, chosen from linear and nonlinear convection-diffusion models.

REFERENCES

- [1] A. Rangarajan, A. Balan, and G. May, Mesh adaptation and optimization for discontinuous Galerkin methods using a continuous mesh model. *AIAA Paper 16-2142*, American Institute of Aeronautics and Astronautics, 2016.
- [2] V. Dolejší, Anisotropic hp -adaptive method based on interpolation error estimates in the L^q -norm. *Appl. Numer. Math.*, Vol. **82**, pp. 80–114, 2014.