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# GEOMETRICALLY NONLINEAR MODES FOR MODEL REDUCTION IN STRUCTURAL DYNAMICS

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This study addresses the development of model reduction methods for geometrically exact beam and shell models. Following the finite element formulations described in [1, 2, 3], the nodal coordinates of the finite element mesh include finite translation and finite rotation variables. As the rotation variables evolve on the group of special orthogonal transformations $SO(3)$, which is a nonlinear space, geometric or Lie group methods have to be considered for space and time discretization.

Model reduction methods often rely on linear projection techniques. Considering that the full-order system is represented by finite element coordinates $\mathbf{q}$ evolving in the linear space $\mathbb{R}^n$, the reduction is indeed based on a *linear transformation* $\mathbf{q} = \mathbf{\Psi}\,\boldsymbol{\eta}$ where $\boldsymbol{\eta} \in \mathbb{R}^{\overline{n}}$ ($\overline{n} < n$) and $\mathbf{\Psi}$ is a rectangular matrix whose columns span the selected reduction subspace. This paradigm is unfortunately not applicable as such to geometrically exact models with rotation variables due to the nonlinearity of the rotation space.

In this work, we study the extension of projection techniques to dynamic systems evolving on a nonlinear space with a Lie group structure, as encountered in geometrically exact models of highly slender structures such as beams and shells. We propose a definition of a *nonlinear projection operator*, which is valid even in the presence of large global and internal motions. The projection operator is defined in the Lie group formalism and maps the system dynamics onto a subgroup of the initial nonlinear space. In this way, the reduced-order model naturally inherits the same Lie group structure as the initial model.

In order to illustrate the behaviour of this nonlinear projection operator, several examples of 3D slender beams undergoing large internal and global motions will be considered.

## REFERENCES

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