

SEGREGATED MULTIGRID PRECONDITIONED METHODS FOR INCOMPRESSIBLE CARDIAC MECHANICS

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From a mechanical perspective, the cardiac muscle behaves as an hyper-elastic, incompressible, orthotropic material. In a finite element framework, the linearization of the incompressible elasticity equations leads to a saddle-point problem of the form

$$\begin{bmatrix} \mathcal{A} & \mathcal{B}^T \\ \mathcal{B} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}. \quad (1)$$

The block \mathcal{A} in (1) (related to the momentum balance equation) is indefinite. This motivates the need of an appropriately fitted strategy for the solution of (1) in Schur-like approaches. To overcome numerical issues related to the numerical solution of (1), in the literature nearly-incompressible descriptions of cardiac behaviour are proposed [1]. Unfortunately, the bulk modulus to use in nearly-incompressible simulations is dependent on the material constitutive law and on the isochoric/deviatoric splitting of the deformation gradient. Adding a positive contribution to this block, we propose novel Segregated Multi-grid Preconditioned (SMGP) methods for the solution of (1). Considering the benchmark problems proposed in [2], the performances of the SMGP methods are assessed for the solution of different aspect in cardiac mechanics. The obtained results show the robustness of the scalability of the proposed method. We compare the results obtained for the incompressible and the nearly-incompressible formulations comparing the results for the three considered problems. For the nearly-incompressible case, we discuss the high dependence of the results on the choice of the bulk modulus.

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