

# First-Order System Least Squares Methods for Coupled Generalized Newtonian Stokes-Darcy Flow

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In this talk the coupling of a generalized Newtonian Stokes flow in one domain and a generalized Newtonian Darcy flow in a porous medium is considered. The formulation is utilizing a first-order system least-squares approach (FOSLS) for the coupled Stokes-Darcy flow. For the Stokes flow a typical stress-velocity formulation, whereas for the Darcy flow a volumetric flux-hydraulic potential formulation, is used. As examples for generalized Newtonian fluids this talk will examine Cross-, Carreau- and Power-law fluids more closely. This approach leads to a nonlinear system of partial differential equations and for the least-squares finite element method to a non-quadratic minimization problem. The coupling is done along an interface where boundary functionals, representing continuity of flux and the balance of forces, are introduced. Additionally the well known boundary condition by Beavers, Joseph and Saffman is used on the interface to ensure well-posedness of the problem. Under some assumptions on the nonlinear viscosity it will be shown that the resulting least-squares functional is an efficient and reliable error estimator which allows for adaptive refinement. Furthermore it will be shown that the arising linear problems in the Gauss-Newton iterations are well posed and the resulting correction does indeed lead to a reduction of the least-squares functional. Stresses and volumetric fluxes will be approximated in  $H(\text{div})$ , using the well known Raviart-Thomas finite element spaces. The structure of the (non-linear) coupled problem in combination with the use of Raviart-Thomas spaces will result in the need of very specific solvers, to achieve scalability of the linear solve with respect to the mesh-size, which will also be investigated in this talk.