

# AN IMPLICIT HIGH-ORDER DISCONTINUOUS GALERKIN METHOD FOR INCOMPRESSIBLE VARIABLE DENSITY FLOWS

F. Massa\*, F. Bassi, L. Botti, A. Colombo

Università degli Studi di Bergamo - Dipartimento di Ingegneria e Scienze Applicate,  
Viale Marconi 5, 24044 Dalmine (BG), Italy  
{francescocarlo.massa,francesco.bassi,lorenzo.botti,alessandro.colombo}@unibg.it

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In this work we present a high-order Discontinuous Galerkin (DG) method for the simulation of variable density incompressible flows. The method is fully implicit and applies to the system of equations comprising the incompressibility constraint, momentum and mass conservation equations. In particular, the density behaves essentially as an advected property describing, in a diffuse fashion, interfaces among multiple fluids. Fluid interfaces are sharply captured by the high-degree polynomial approximation, thus not requiring a geometrical reconstruction while ensuring mass conservation.

The distinguishing feature of the proposed formulation is the inviscid interface flux, which is based on the solution of local Riemann problems perturbed by means of an artificial compressibility term [1]. The resulting formulation perfectly fits within a DG framework and the tight coupling between pressure and velocity stabilizes the method and allows equal-order approximation for both pressure and velocity. Time integration is performed by means of implicit schemes.

Several numerical aspects related to the implementation of the present approach are investigated. Among them, the use of different sets of working variables, *e.g.* logarithms and hyperbolic functions, to control density over/undershoots, and the treatment of spurious density oscillations at flow interfaces by the addition of a local artificial viscosity term to the discretization [2, 3]. Both these actions greatly improved the code robustness especially when dealing with high density ratios, *e.g.*, water–air.

Promising results on numerical experiments involving also high-density ratios and interactions of more than two fluids have been obtained using very high-order polynomial approximations on relatively coarse grids.

## REFERENCES

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