

## A level-set approach for a multiscale cancer invasion model

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We present a multiscale model for tumor invasion and its implementation with adaptive finite elements using cut cells in a two dimensional domain. In particular, we show a new formulation, based on the level-set method, for the model presented in [1]. The macroscopic dynamics determines the distribution of cancer cell  $c$  and extracellular matrix  $v$  in the domain  $\Omega(t)$ , see equations (1–2). The time-dependent domain of the cancer region is modelled as the zero-set of an initial level-set function  $\phi$  that is transported according to a computed velocity field, see equation (3). The interface dynamics is determined by the solution of a microscopic quantity  $m$  (distribution of matrix degrading enzymes) at the boundary of the cancer region, see equation (4). We show numerical results and discuss possible extension of the model.

Macroscopic model component:

$$\frac{\partial c}{\partial t} = D_1 \Delta c - \eta \nabla \cdot (c \nabla v) + \mu_1 c(1 - c - v) \quad \text{in } \Omega(t) \times [0, T] \quad (1)$$

$$\frac{\partial v}{\partial t} = -\alpha c v + \mu_2(1 - c - v) \quad \text{in } \Omega(t) \times [0, T] \quad (2)$$

Transport of domain boundary:

$$\frac{\partial \phi}{\partial t} + v(m) \cdot \nabla \phi = 0, \quad \text{in } \Omega' \times (0, T]. \quad (3)$$

Microscopic model component:

$$\frac{\partial m}{\partial t}(y, t') = D_2 \Delta m(y, t') + F_x(c) \quad (y, t') \text{ in } \epsilon Y \times [0, T'] \quad (4)$$

## REFERENCES

- [1] D. Trucu, P. Lin, M. A. Chaplain, and Y. Wang. A multiscale moving boundary model arising in cancer invasion. *Multiscale Modeling & Simulation*, 11(1):309–335, 2013.