

Flow and Mechanics in Fractured Media as Mixed-Dimensional PDEs

Wietse M. Boon^{1*}, Jan M. Nordbotten²

¹ University of Stuttgart, Pfaffenwaldring 61, Stuttgart, wietse.boon@iws.uni-stuttgart.de

² University of Bergen, Allégaten 41, Bergen, jan.nordbotten@uib.no

Key Words: *Instructions, Multiphysics Problems, Applications, Computing Methods.*

In the mixed-dimensional representation of fractured media, the fractures are considered as lower-dimensional manifolds. This concept is successively applied to the lines and points at the intersections between fractures leading to a hierarchical geometry of manifolds of codimension one. By imposing these modelling assumptions a priori in the continuous setting, the basis is formed for the introduction of coupled PDEs on the mixed-dimensional geometry, which we refer to as mixed-dimensional PDEs.

In this work, we consider Darcy flow combined with linear elasticity on mixed-dimensional representations of fracture networks. Since the associated, governing equations are fully coupled, the systems of equations are presented using mixed-dimensional differential operators which map between the different dimensions. In turn, the resulting system of equations is considered as mixed-dimensional and is analysed as such, before the introduction of the discretization scheme.

We present theoretical results related to the structure of mixed-dimensional elliptic partial differential equations from which multiple conforming discretization schemes arise using dimensionally hierarchical finite elements.

Keeping later purposes such as transport problems and fracture propagation in mind, our main interest lies in obtaining accurate flux fields and stress states which respect physical conservation laws. Therefore, we employ mixed finite elements which allow for a local preservation of such laws. The symmetry of the stress tensor is imposed in a weak sense, thus leading to the use of familiar, conforming, finite elements with relatively few degrees of freedom.

Results concerning convergence and stability of the mixed finite element schemes are shown. These are supported by numerical examples in two- and three-dimensional domains in which the lower-dimensional inclusions, intersection lines, and points have significantly different material properties compared to the surroundings.