

# A semi-analytical solution for the problem of a load moving in an underground tunnel

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Train-induced vibration has been an important issue in the past three decades due to the continuous development of ground/underground trains. The methods proposed to predict this type of vibration can be subdivided into two categories: numerical methods (e.g., [1]) and analytical methods (e.g., [2]). Numerical models are often computationally expensive but can deal with complex geometries. Analytical methods are computationally efficient but are limited to structures of simple geometries. This study aims at presenting a semi-analytical solution describing the train-induced vibration for a shallow tunnel embedded in an elastic half-space by employing the complex-variable theory.

First, the Fourier transform is applied over time and the longitudinal co-ordinate of the tunnel. Thereafter, we use the following methods to solve the formulated boundary value problem: 1) Two conformal mapping functions are employed to transform the original domain with two symmetries (cylindrical and translational) into a mapped domain with only one symmetry (cylindrical), which makes the problem easier to solve; 2) The reflected waves generated at the free surface are represented as cylindrical waves propagating from the image of the tunnel, considered with respect to the free surface; 3) An orthogonality relation is applied to calculate the unknown coefficients. Lastly, the inverse Fourier integrals over frequency and wavenumber are evaluated to get the time-space domain response.

Some numerical examples are given, and results are validated with those in reference [2]. The current model can also be coupled to a detailed train-track model. Furthermore, the equivalent stiffness of the half-space with an embedded tunnel can be also derived, which allows to study the stability of an oscillator moving in the tunnel.

## REFERENCES

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