

## EFFECTIVE MODEL REDUCTION FOR SHALLOW WATER FLOWS

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### ABSTRACT

The Shallow Water Equations (SWE) are widely used across many fields. Despite their frequent application, efficient solution of the SWE remains a challenge under many conditions. One approach to alleviate the computational burden in applications is to use formal model reduction. Among the many available techniques, Proper Orthogonal Decomposition (POD)-based methods [1, 2] are widely popular, since they are relatively straightforward to formulate while maintaining a rigorous connection to the underlying high-fidelity model.

Unfortunately, direct application of POD techniques can fail to be robust or efficient for complex flows. One challenge is the need to approximate (potentially non-polynomial) nonlinearities accurately in a reduced model. Moreover, it can be difficult to compress solutions of the SWE efficiently when the solution exhibits shocks and complex wave behavior. The first challenge can be addressed through appropriate use of hyper-reduction [3, 4]. For the second, we follow a recent approach from [5] and switch to a Lagrangian frame of reference, where wave-like solutions can have a lower rank structure.

Here, we consider the effectiveness of these approaches for reduction of dam-break and riverine flows. As a point of reference, we consider a class of non-intrusive methods based on combining POD and Radial Basis Function (RBF) interpolation [6, 7]. We evaluate the methods in terms of their accuracy, computational expense, and robustness across test problems.

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