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Multilevel Monte Carlo methods for Uncertainty Quantification and robust design optimization

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Complex mathematical models, based on partial differential equations, are widely used in many areas of physics and engineering. However, it is often the case that some of the parameters entering those models are affected by uncertainty either due to an intrinsic variability or lack of knowledge. Forward Uncertainty Quantification aims at propagating the input uncertainty, often described in probabilistic terms, through the model and quantifying the corresponding uncertainty in its solution or output quantities of interest.

Monte Carlo methods, although robust and simple to implement, often lead to unaffordable computational costs in this setting as they require an excessive number of forward runs of the complex model to achieve acceptable tolerances.

The multilevel Monte Carlo method has proven to be very powerful to compute expectations of output quantities of a stochastic model governed by differential equations. It exploits several discretization levels of the underlying equation to dramatically reduce the overall complexity with respect to a standard Monte Carlo method.

In this talk we review the main ideas of the Multilevel Monte Carlo method and discuss practical implementation aspects as well as extensions to accommodate concurrent types of discretizations (multi-index Monte Carlo method) and compute derived quantities such as central moments, quantiles, or cdfs of output quantities. We illustrate the power of the MLMC method on applications such as compressible aerodynamics, shape optimization under uncertainty, ensemble Kalman filter and data assimilation.

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