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Optimal preconditioners of linear complexity for problems of negative order discretized on locally refined meshes

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Discretized operators of negative order arise by the application of the boundary element method, or as Schur complements in domain decomposition methods. Using a boundedly invertible operator of opposite order discretized by continuous piecewise linears, we construct an optimal preconditioner for operators of negative order discretized by (dis)continuous piecewise polynomials of arbitrary order. Our method ([5]) is a variation of the well-studied dual mesh preconditioning technique [4, 3, 1]. Compared to earlier proposals, it has the advantages that it does not require the inverse of a non-diagonal matrix, it applies without any mildly grading assumption on the mesh, and it does not require a barycentric refinement of the mesh underlying the trial space.

The cost of the preconditioner is the sum of the cost of the discretized opposite order operator plus a cost that scales linearly in the number of unknowns. Thinking of the canonical example of the single layer operator, an obvious choice for the operator of opposite order is the hypersingular operator. Aiming at a preconditioner of optimal complexity, however, we wish to apply a multi-level operator instead, as the one proposed in [2].

Other than with operators of positive order, so far on locally refined meshes optimal (multi-level) preconditioners of linear complexity seem not to be available. We modify the method from [2] such that it applies on locally refined meshes.

REFERENCES

[1] A. Buffa and S.H. Christiansen. A dual finite element complex on the barycentric refinement. *Math. Comp.*, 76(260):1743–1769, 2007.

[2] J. H. Bramble, J. E. Pasciak, and P. S. Vassilevski. Computational scales of Sobolev norms with application to preconditioning. *Math. Comp.*, 69(230):463–480, 2000.

[3] R. Hiptmair. Operator preconditioning. *Comput. Math. Appl.*, 52(5):699–706, 2006.

[4] O. Steinbach and W. L. Wendland. The construction of some efficient preconditioners in the boundary element method. *Adv. Comput. Math.*, 9(1-2):191–216, 1998. Numerical treatment of boundary integral equations.

[5] R.P. Stevenson and R. van Venetië. Optimal preconditioning for problems of negative order, 2018, arXiv:1803.05226. Submitted.