

## **ANALYTICAL DEFINITION OF THE CRITICAL LOAD OF A LONG-REINFORCED CONCRETE PIPE WITH VARIABLE GEOMETRY**

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An analytical approach based on the Rayleigh method is adopted to calculate the buckling load of an actual 46-m-high concrete pipe, taking into account the geometric stiffness, functions of the concentrated forces, and self-weight of the structure. Stability analysis that included the self-weight of structural elements was originally discussed by Euler (1774), but he did not succeed in obtaining a satisfactory solution. The problem was finally resolved by Greenhill in 1881 [1].

The pipe is made of concrete and its geometric nonlinear behavior and imperfections are linearized by reducing the structural stiffness, as can see in [2] and [3]. The material is considered to be viscoelastic with reduction of the flexural stiffness, [4], to consider the material non-linearity and creep is taken into account by the criteria from Eurocode 2 [5]. The ground is modeled as a set of distributed springs.

Then, the critical buckling load is calculated analytically and dynamically defined to different instants along time after the structure to be loaded. Modulus of elastic and specific deformation on time are also obtained. Finally, the structural stiffness is evaluated. Reductions of 25.15% to the modulus of elasticity and deformation, and of 21.08% to the critical load of buckling for analysis performed at zero and four thousand days were found.

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