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An explicit high-order residual distribution scheme for time-dependent hyperbolic systems

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One of the major difficulties when solving a system of multidimensional time-dependent partial differential equations in an explicit way using finite element type techniques, is the treatment of the so-called mass matrix. Indeed, using standard solvers, one needs to invert the mass matrix given by the integral over the considered basis functions belonging to the term of the partial derivative in time.

The proposed scheme intends to avoid any of this related issues, as it allows for an efficient diagonalization of the mass matrix without any loss of accuracy. This has been possible via a specifically designed time-stepping approximation which has been inspired by Deferred Correction based methods [1,2]. We allow to solve our system explicitly and are able to guarantee high-order of accuracy both in space and time.

More precisely, the leading idea is based on using a prediction correction method, where the number of corrections is directly proportional to the desired order of accuracy.

Further, spatial terms are approximated with a Residual Distribution scheme [3], which can be reinterpreted as a Discontinuous Galerkin (DG) like method and has some of its features, as making use of the same quadrature approximations, being easily extended to unstructured meshes and having a local stencil. Contrarily to DG methods, RD allows for a slower growth of degree of freedoms, required when interested in enhancing the accuracy of the method.

Initially developed for scalar equations [4], this presented novel approximation method is extended to multidimensional systems of equations [5].

We have assessed our method on several challenging benchmark problems for one- and two-dimensional Euler equations and the proposed scheme has proven to be robust and to achieve the theoretically predicted high order of accuracy on smooth solutions.

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