

On analysis and discretization of operator functions

Christian Engström¹

¹ Umeå University, Department of Mathematics and Mathematical Statistics
SE 901 87 Umeå, Sweden

christian.engstrom@umu.se

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Functions whose values are linear differential operators and block operator matrices are used to model e.g. fluid-solid interactions, vibrations of rotating beams, dissipative acoustics, and propagation of electromagnetic waves.

In this talk, we consider both finite element approximations of the eigenvalues of operator functions and quantitative properties of the discrete spectrum. Properties of the discrete spectrum of non-selfadjoint Schrödinger operators have been studied intensively since Pavlov's influential papers in the 1960s on accumulation of complex eigenvalues to the essential spectrum. Rational operator functions share spectral properties with a linear non-selfadjoint block operator called the linearized operator. However, since in non-trivial cases the linearized operator is not a relatively compact perturbation of a selfadjoint operator, the known results for non-selfadjoint operators cannot be applied to rational (or polynomial) eigenvalue problems.

We will briefly discuss how sufficient conditions for accumulation of branches of eigenvalues can be derived for rational eigenvalue problems. Then, Galerkin approximations of these eigenvalues are considered and difficulties that arise when the operator function is not Fredholm-valued is discussed. This talk is based on [1-4] and ongoing work on fluid-solid interactions and on dissipative acoustics.

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