

CONTEMPORARY CHALLENGES AND PROBLEMS OF FRACTIONAL CALCULUS IN APPLIED MATEMATICAL MODELLING OF COMPLEX PROCESSES IN NATURE

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Mathematical description of complex processes and phenomena is an attractive field for sciences. Unlike to simple processes, the complex ones are identified as multiscale processes, instantaneous internal structures inside them, mutual relationship between coefficients and arbitrary parameters. In this case, fractional calculus [4,5,6] as a mathematical tool may help scientists for better illustration of known rules in application to description of such complexity. Some fundamental studies has been done in [1,2,3,4,5]. Some mathematical and approach problems/obstacles rise from such application. Here we identify the most important as: improper formulation of a set of equations; inappropriate units of measure occurring in algebraic equations; improper formulation of initial-boundary conditions.

Against this background we would like to show how to solve problems listed above. I.e. formulation of proper set of equations must include different approach concerning expansion of the function as presented by the Fig. 1.

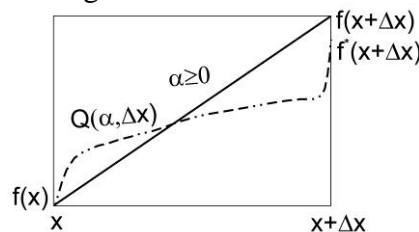


Fig. 1: Function expansion in non-linear form $Q(\alpha, \Delta x)$.

Using this expansion, we show how to illustrate the principles and laws existing in mechanics, as: continuity, momentum and energy equations being useful for modelling complex processes.

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