

## Controlling eutrophication by water artificial circulation

Francisco J. Fernández<sup>1</sup>, Aurea Martínez<sup>2</sup> and Lino J. Alvarez-Vázquez<sup>3</sup>

<sup>1</sup> Centro Universitario de la Defensa, Escuela Naval Militar, 36920 Marín, Spain,  
fjavier.fernandez@cudefensadefensa.es, <https://gscpage.wordpress.com/about/>

<sup>2</sup> Depto. Matemática Aplicada II, Universidade de Vigo, E.I. Telecomunicación 36310  
Vigo, Spain, aurea@dma.uvigo.es, <http://www.dma.uvigo.es/~aurea/>

<sup>3</sup> Depto. Matemática Aplicada II, Universidade de Vigo, E.I. Telecomunicación 36310  
Vigo, Spain, lino@dma.uvigo.es, <http://www.dma.uvigo.es/~lino/>

**Keywords:** *Optimal control, Partial differential equations, Eutrophication, Water artificial circulation, Restoration*

This work deals with artificial circulation as a shallow water aeration technique. Large waterbodies (for instance, lakes or reservoirs) get much of their oxygen from the atmosphere through diffusion processes. Artificial circulation increases water's oxygen by forcefully circulating the water to expose more of it to the atmosphere. Two techniques are the most common: air injection and mechanical mixing. The former has been analyzed from an ecological viewpoint in several works (see, for instance, [1] and the references therein). However, in this work we will focus our attention on the latter that, as far as we know, has remained unaddressed in the mathematical literature.

In this work we will introduce a mathematical formulation of the environmental problem as a control/state constrained optimal control problem of partial differential equations. Then, we will analyze the optimal control problem and finally, we will deal with the numerical resolution of the problem, presenting a complete numerical algorithm and a realistic computational example [2].

This work was supported by funding from project MTM2015-65570-P of Ministerio de Economía y Competitividad (Spain)/FEDER.

### REFERENCES

- [1] R.C. HAYNES, *Some ecological effects of artificial circulation on a small eutrophic lake with particular emphasis on phytoplankton*, *Hydrobiologia*, **43** (1973), 463-504.
- [2] A. MARTÍNEZ, F.J. FERNÁNDEZ AND L.J. ALVAREZ-VÁZQUEZ, *Water artificial circulation for eutrophication control*, *Math. Control Rel. Fields*, (2017), in press.