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Mixed-dimensional coupled finite elements in FEniCS

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Mixed-dimensional partial differential equations (PDEs) are equations coupling unknown fields defined over domains of differing topological dimension. Such mixed-dimensional PDEs naturally arise in a wide range of fields including geology, bio-medicine, and fracture mechanics. In the brain, the blood vessels and paravascular spaces play a important role in the circulation, flow and exchange of tissue fluid. This process can be investigated via a mathematical approach based on coupled mixed-dimensional models mimicking the vasculature as topologically one-dimensional structures embedded in a three-dimensional porous medium.

The FEniCS project [1] aims at automating the numerical solution of mathematical models based on PDEs using finite element methods, and is organized as an open source collection of software components. A core feature is a high-level domain-specific language for finite element spaces and variational forms close to mathematical syntax.

Finite element discretizations of mixed-dimensional PDEs involve nested meshes of heterogeneous topological dimension and typically require specific averaging operators. The assembly of such systems is non-standard and non-trivial especially with regard to the terms involved in the interactions between the different domains. In other words, automated solution of mixed-dimensional PDEs requires the design of both generic high level software abstractions and lower level algorithms.

In this talk, we present a set of features developed as part of an automated framework dedicated to such problems in the FEniCS Project framework. The introduced tools will be illustrated by concrete examples of applications highlighting their relevance for model in biomedicine in general and for the brain's waterscape in particular.

REFERENCES

[1] M. Alnæs, et al. *The FEniCS Project Version 1.5.* Archive of Numerical Software 3.100 (2015): 9-23. *www.fenicsproject.org*

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