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OPTIMIZATION OF DISPERSIVE COEFFICIENTS FOR WAVE PROPAGATION IN TWO-PHASE PERIODIC STRUCTURES

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The purpose of this talk is to report on a recent work [1], devoted to dispersive effects of wave propagation in periodic media, which can be modelled by adding a fourth-order term in the homogenized equation (see [3] for the first formal derivation of this model and [2] for the first rigorous mathematical proof of convergence). Dispersion is classically defined as the phenomenon by which waves with different wavelengths propagate with different velocities. More precisely, for a small period $\epsilon > 0$, we consider the following scalar wave equation

$$\frac{\partial^2 u_\epsilon}{\partial t^2} - \operatorname{div}\left(a\left(\frac{x}{\epsilon}\right)\nabla u_\epsilon\right) = 0, \tag{1}$$

with periodic coefficients and initial data $u_\epsilon(0,x) = u^{\text{init}}(x)$, $\frac{\partial u_\epsilon}{\partial t}(0,x) = v^{\text{init}}(x)$. According to [3], [2], it can be approximated up to long times of order ϵ^{-2} by the following high order homogenized equation

$$\frac{\partial^2 v_\epsilon}{\partial t^2} - \operatorname{div}\left(a^*\nabla v_\epsilon\right) + \epsilon^2 \mathbb{D}^*\,\nabla^4 v_\epsilon = 0. \tag{2}$$

where the fourth-order dispersive tensor $\mathbb{D}^*$ is called Burnett tensor. We numerically optimize the coefficients of $\mathbb{D}^*$ in order to minimize or maximize dispersion. We restrict ourselves to the case of a two-phase composite medium with an 8-fold symmetry assumption of the periodicity cell in two space dimensions. We obtain numerical upper and lower bound for the dispersive properties, along with optimal microgeometries.

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