

## NEUMANN BOUNDARY CONDITIONS FOR HIGH-ORDER MODAL DG-SCHEMES

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A simple homogeneous Neumann boundary condition can be achieved by a straightforward extrapolation of the state in the Discontinuous Galerkin (DG) scheme. However, we found this approach to result in instabilities with our high-order modal DG implementation at subsonic inflow boundaries of the inviscid, compressible flows described by the Euler equations.

While there exist sophisticated boundary conditions that alleviate this problem, see for example [1] for a general discussion and [2] for a specific approach in DG, those are more involved and require the consideration of the evolution of the boundaries in time. We consider here a special approach tailored towards the modal implementation of the DG scheme and stick to a extrapolation scheme without the need for special boundary elements. We employ the orthonormal basis of Legendre polynomials in this scheme, which allows us to find a extrapolation, where the resulting polynomial has no gradient at the boundary.

We consider the polynomial representation of the state normal to the boundary in terms of a Legendre expansion and modify the last mode to achieve a zero gradient at the boundary. Due to the properties of the Legendre polynomials this can be easily computed and a value is found that can be considered as an extrapolation of the state without a gradient at the boundary.

We show how this minimal change in the boundary condition helps to stabilize the inflow boundaries in this scheme for high-order approximations with 32 modes and more in the boundary normal direction.

### REFERENCES

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