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# 'On the fly' snapshot selection for hyper-reduced Proper Orthogonal Decomposition with application to nonlinear dynamic.

**P. PHALIPPOU[1], S. Bouabdallah[2], P. Breitkopf[3] and P. Villon[4]**

[1] Altair Engineering France, Centre d'affaires - Bâtiment C, 5/10 Rue de la Renaissance, 92184 Antony Cedex, pphalippou@altair.com
[2] Altair Engineering France, Centre d'affaires - Bâtiment C, 5/10 Rue de la Renaissance, 92184 Antony Cedex, sbouabdallah@altair.com
[3] Laboratoire Roberval, Université de Technologie de Compiègne, piotr.breitkopf@utc.fr
[4] Laboratoire Roberval, Université de Technologie de Compiègne, pierre.villon@utc.fr

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Industrial usage of numerical math-based tools such as the Finite Element Method (FEM) may, in some applications, become prohibitive due to the computational cost. This issue is particularly real in the automotive sector when optimizing the shape of a vehicle in crash situations. Model Order Reduction (MOR) addresses this issue.

The Hyper-Reduced Proper Orthogonal Decomposition (HRPOD) [5] consists in three phases. During the observation phase data is collected from runs of the Full Order Model (FOM). The Reduced Order Model (ROM) is then post-processed from the gathered data. It consists of a Reduced Basis (RB) $[\Phi] \in \mathbb{R}^{N\times k}$ and a reduced cubature scheme for internal forces integration. Finally, the system of discrete equations is projected on the RB. The two first phases are referred to as the training phase and may be performed off-line while the last one is the on-line phase.

The general discretized FOM used in nonlinear structural dynamic writes

$$[\mathbb{M}]\{\ddot{u}(t)\} + [\mathbb{C}]\{\dot{u}(t)\} + F^{NL}_{int}(\{u(t)\}, t) = F_{ext}(t)$$

The displacement $\{u(t)\} \in \mathbb{R}^N$ is approximated by $\{\tilde{u}(t)\} = [\Phi]\{\alpha(t)\} + \{\bar{u}\}$. $\{\bar{u}\}$ is a lift used to center observations in the POD. Injecting this approximation into the FOM and projecting on the RB gives the ROM.

$$[\tilde{\mathbb{M}}]\{\ddot{\alpha}(t)\} + [\tilde{\mathbb{C}}]\{\dot{\alpha}(t)\} + [\Phi]^T F^{NL}_{int}(\{\tilde{u}(t)\}, t) = [\Phi]^T F_{ext}(t)$$

Where $[\tilde{\mathbb{M}}] = [\Phi]^T[\mathbb{M}][\Phi]$ and $[\tilde{\mathbb{C}}] = [\Phi]^T[\mathbb{C}][\Phi]$. $\{\alpha(t)\} \in \mathbb{R}^k$ are the degrees of freedom of the ROM with $k << N$.

Various issues such as nonlinear internal forces integration, contact reduction, and reduced model training cost arise from the application of HRPOD to the optimization process. The '*offline*' computational cost of training the ROM is an essential bottleneck in industrial applications. This presentation addresses this last concern.

One lead to crushing the off-line training phase complexity lies in incremental training methods similar to the incremental Singular Value Decomposition (SVD) [1]. The underlying idea in incremental methods in MOR is to post-process observations as soon as available, providing the numerous advantages. Expensive read/write in binaries are avoided. As individual observations contribution to the ROM may be evaluated ahead of processing them, useless computations are avoided.

However, difficulties appear in the use of the incremental SVD within the HRPOD framework. Numerical noise needs to be correctly handled. To ensure the incremental SVD stability the RB needs to be kept orthonormal, but subsequent reorthonormalizations may deteriorate precision and drastically lengthen computation time. The '*a-posteriori*' centering of the resulting RB [4] is in this work shown to be inefficient as it is not equivalent to '*a priori*' centering and deteriorate the RB. RB error of approximation is not accessible as observations are not kept in memory. An error estimator is mandatory to drive the method. Last but not least it may be necessary to keep some observations in memory if they are used by another complementary reduction method, for example, clustering, that does not have an incremental counterpart yet.

The HRPOD with incremental RB training has been implemented in ALTAIR's solver RADIOSS© and applied to dynamic test cases. This presentation focuses on the incremental RB training. A criterion to handle numerical noise is proposed along with a method to incrementally center observations within the training phase. The contribution of centering the observations is discussed. A new error of approximation estimator is developed to handle 'on the fly' truncation in the incremental SVD. To conclude, the necessary developments to make the whole training phase incremental are discussed.

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