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Invariance of Mechanical Quantities Regarding Eigenvector Functions Obtained from Computational Stability Analysis of Structures

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It is shown that the absolute value υ_0 of the initial vector velocity $(\mathbf{r}_1')_0$ and the ratio a/a_0, where a is the absolute value of the vector acceleration $\mathbf{r}_1''$ and a_0 is its initial value, are invariant quantities regarding eigenvector functions $\mathbf{r}_1(t)$. ($\mathbf{r}_1'$ and $\mathbf{r}_1''$ denote the first and the second derivative of $\mathbf{r}_1$ with respect to the pseudo time *t*, with $d\,t = (d\mathbf{q}\cdot d\mathbf{q})^{\frac{1}{2}}$, where $d\mathbf{q}$ is a differential of the nodal displacement vector, $\mathbf{q}$, in the framework of the Finite Element Methode (FEM).) The functions $\mathbf{r}_1(t)$ represent special eigenvector functions, obtained from linear eigenvalue problems, with the tangent stiffness matrix $\tilde{\mathbf{K}}_T(t)$ and an arbitrary real symmetric matrix **B** as the coefficient matrices. At the stability limit, these functions are related to the null eigenvalue.

The proof of the asserted invariance is based on the reduction of N-dimensional normalized eigenvectors to 3-dimensional eigenvectors, the vertices of which represent curves on an octant of the unit sphere. The terms “vector velocity” and “vector acceleration” refer to a fictitious particle, moving with variable speed on these surface curves. Thus, by “invariance of mechanical quantities regarding eigenvector functions” the independence of υ_0 und a/a_0 of the matrix **B** is meant. The mechanical background of these invariances is the invariance of $(U-U_M)/U = 2\upsilon_0(a/a_0)$, where U denotes the strain energy and U_M stands for its membrane part. The relation between the kinematic quantity $2\upsilon_0(a/a_0)$ and the energetic quantity $(U-U_M)/U$ was previously derived for the so-called, “consistently linearized eigenvalue problem”, characterized by $\mathbf{B} = \tilde{\mathbf{K}}_T'$ [1].

REFERENCE

[1] H.A. Mang, Evolution and verification of a kinematic hypothesis for splitting of the strain energy. *Comput. Methods Appl. Engrg.*, Vol. **324**, pp. 74-109, 2017.