

Hybridized SLAU2-HLLI for Magnetohydrodynamics Simulations

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SLAU2^[1], categorized as AUSM-type Riemann solvers, has been extensively developed in gasdynamics. It is based on a splitting of the numerical flux into advected and pressure parts. In this paper, this Riemann solver has been extended to MHD. The SLAU2 Riemann solver has the favorable attribute that its dissipation for low speed flows scales as $O(M^2)$, where “ M ” is the Mach number, as required for low speed flows. At higher Mach numbers, however, the pressure-split Riemann solver was found not to function well for some MHD Riemann problems, despite the fact that they were engineered to have a dissipation that scales as $O(|M|)$ for high Mach number flows.

The HLLI Riemann solver^[2] has a dissipation that scales as $O(|M|)$, which makes it unsuitable for low Mach number flows. However, it has very favorable performance for higher Mach number MHD flows. Since the two families of Riemann solvers both perform very well over a range of intermediate Mach numbers, the best way to benefit from the mutually complementary strengths of both these Riemann solvers is to hybridize between them. The result is an all-speed Riemann solver for MHD. We, therefore, document hybridized SLAU2-HLLI Riemann solver.

$$\mathbf{F}_{SLAU2-HLLI} = \chi \mathbf{F}_{SLAU2} + (1 - \chi) \mathbf{F}_{HLLI} \quad (1a)$$

$$\chi = (1 - \hat{M}')^2, \quad \hat{M}' = \min \left(1.0, \max \left(\frac{K}{\bar{c}} \sqrt{\frac{\mathbf{u}_L^2 + \mathbf{u}_R^2}{2}} - M_{co}, 0 \right) \right) \quad (c: \text{fast magnetosonic speed}) \quad (1b)$$

where $K=4.0$ (i.e., HLLI is used entirely for $M > 0.25$), and $M_{co} = 0.1$ so that this solver turns to a full “SLAU2” for $M < 0.1$. The hybrid Riemann solver suppresses the oscillations (Fig. 1) that appeared in single solver solutions, and it also preserves contact discontinuities, as well as Alfvén waves, very well. Furthermore, its better resolution at low speeds has been demonstrated. We also present several stringent one-dimensional test problems.

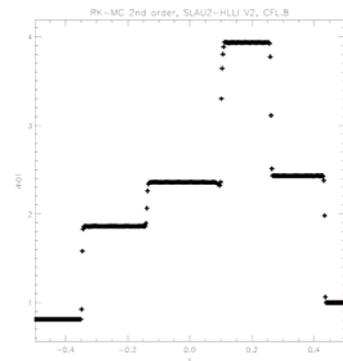


Fig. 1 Shocktube problem solution (density).

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