

# A GLOBALLY NINTH-ORDER STABLE FINITE DIFFERENCE SCHEME AND ITS APPLICATIONS TO TWO-DIMENSIONAL PROBLEMS

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To retain the formal order of a high-order scheme for first-order hyperbolic problems, boundary closures must be at most one order lower than the interior scheme according to Gustafsson's result [1]. For very high-order interior schemes, however, it is a challenge to derive stable boundary closures to achieve this. There may be a contradiction between satisfying the uniform high-order condition and keeping the schemes strictly stable (also called time stable), which is very important for long time simulations [2].

In this talk, we present a new stable boundary closures for a ninth-order dissipative compact finite difference scheme. We will first show that boundary closures derived on uniform grids are not strictly stable when a globally ninth order of accuracy is retained. Then we will introduce a globally conservative boundary operator and redetermine the near boundary solution points, which are finally nonuniformly distributed. Using this new solution points, we show that the scheme is globally conservative. In addition, the eigenvalues of the coefficient matrix of the new algorithm all distribute in the left half complex plane, indicating the strict stability property of the scheme. To demonstrate both the stability and the design order of the scheme, some benchmark problems governed by two-dimensional Euler equations are presented, including a vortex transport, a radial expansion wave, a channel flow and the Ringleb problem. The results show the excellent stable property of the new boundary closures.

## REFERENCES

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