

# On the micromechanical foundations of the anisotropy of volumetric growth in soft biological tissues

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Various biological tissues change their shape by so-called volumetric growth, where cells deposit material (e.g., fibers) in the interior of the tissue. Over time, this deposition results in significant changes of the tissue geometry, which play key roles during various life stages from morphogenesis to aging.

Theoretical and computational models of the growth of living soft tissues have attracted rapidly increasing attention over the last decade [1-5]. Nowadays, much of the work in this area is – in some or another way – based on a split of the deformation gradient in an inelastic part representing growth and an elastic part that ensures mechanical equilibrium and geometric compatibility. Most of the time, the inelastic growth part of the deformation gradient is chosen in a rather heuristic way. If not much is known about the growth process, it is typically chosen as an isotropic tensor. In some cases where experimental observations are available simple anisotropic tensors such as transversely isotropic tensors are used. To overcome these rather heuristic approaches, in [6], it was suggested to determine the inelastic growth tensor based on an Eshelby-like stress tensor and a dissipative principle. Despite its theoretical elegance this approach suffers from certain limitations. In particular, it is based on parameters which have significant impact on the growth anisotropy but may be hard to determine for predictive computations in soft biological tissues.

Herein, we develop a simple multi-scale model of volumetric growth that naturally predicts the anisotropy of the growth dynamics as a function of the tissue stiffness. We demonstrate how one can compute the volumetric growth tensor at each point in the tissue and show that its predictions are in good qualitative agreement with the observed growth in a host of different biological tissues. This gives rise to the hope that the presented novel theoretical framework could form an important step towards a mathematically rigorous answer to the open question of how to compute the inelastic deformation gradient describing volumetric growth in soft biological tissues.

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