

# COMPREHENSIVE KINEMATICS AND KINETICS OF COSSERAT BEAMS AND THEIR APPLICATION FOR DEVELOPING MEASUREMENT MODEL FOR STRAIN GAUGES

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The idea of modelling a beam subjected to large deformation by means of framed curve is attributed to Duhem and the Cosserat brothers. One of the prime reasons for the successful acceptance and continuous research on Cosserat beam theory is its mathematical simplicity in modeling a rod-like structure as a single manifold problem. Our research on obtaining global deformed shape of rod-like structure has lead us to investigate and improve the kinematics of Cosserat beams subjected to large deformations.

In this work, we present an exhaustive, geometrically-exact, non-linear kinematics model that captures deformation due to multiple curvatures, torsion, shear, axial deformation and coupled Poissons and warping effects. We discuss an attempt to capture the coupling between Poissons effect and warping and detail a unique way to capture 3D shear deformation in the beam. We delineate the inconsistencies and challenges that are encountered because of our ambition to obtain an exhaustive kinematic model that includes the out-of-plane deformation of the cross-section, all the while maintaining the single manifold nature.

We develop the deformation gradient tensor and strain vector for the beam and carefully detail the contribution of various deformation effects to the strain vector. The Cauchy-Green deformation tensor can be considered as a Push-Forward Riemannian metric. The Push-Forward Riemannian metric helps us investigate the relationship between the scalar strain measurement of the finite-length strain gauge and the local finite strain parameter. The kinematics developed is used to establish a measurement model of discrete and finite length strain gauges attached to the surface of the beam or embedded into the beam.

We briefly discuss about the balance laws associated with Cosserat beam and interpret the DAlembert forces from the non-inertial director frame of reference. The discussion on the energy conservation law and the condition associated with it is discussed and the relation between the Lagrangian and Hamiltonian functional for the Cosserat beam is obtained.

The mechanics developed here offers a method to obtain strain parameters using the sur-

face measurements which is directly useful to develop a shape reconstruction methodology of slender structures. The comprehensive nature of the kinematics has potential to perform accurate analysis and capture 3D deformation (including shear and warping) just by modeling the structure as a space curve.