6th European Conference on Computational Mechanics (ECCM 6)
7th European Conference on Computational Fluid Dynamics (ECFD 7)
11-15 June 2018, Glasgow, UK

SPLINE FUNCTIONS, THE DISCRETE BIHARMONIC OPERATOR, AND APPROXIMATE EIGENVALUES

Matania Ben-Artzi[1], **Guy Katriel**[2]

[1] Institute of Mathematics, The Hebrew University, Jerusalem 91904, Israel,
mbartzi@math.huji.ac.il

[2] Department of Mathematics, Ort Braude College, Karmiel 21982, Israel,
katriel@braude.ac.il

Keywords: *Biharmonic problem, Discrete Biharmonic Operator, Splines, Eigenvalues*

The biharmonic operator plays a central role in a wide array of physical models, notably in elasticity theory and the streamfunction formulation of the Navier-Stokes equations. The need for corresponding numerical simulations has led, in recent years, to the development of a discrete biharmonic calculus. The primary object of this calculus is a high-order compact discrete biharmonic operator (DBO). The numerical results have been remarkably accurate, and have been corroborated by some rigorous proofs. However, there remained the "mystery" of the "underlying reason" for this success. Our work is a contribution in this direction, showing the strong connection between cubic spline functions (on an interval) and the DBO. It is shown in particular that the (scaled) fourth-order distributional derivative of the cubic spline is identical to the action of the DBO on grid functions. A remarkable consequence is the fact is that the kernel of the inverse of the discrete operator is (up to scaling) equal to the grid evaluation of the Green's function of the biharmonic operator, providing an explicit expression for the matrix of the DBO. We use these results to study the relation between the (infinite) set of eigenvalues of the fourth-order biharmonic operator on an interval and the finite set of eigenvalues of the discrete biharmonic operator. The discrete eigenvalues are proved to converge (at an "optimal" $O(h^4)$ rate) to the continuous ones.