

# Cahn-Hilliard Phase Field Approach for Adjoint Two-Phase Fluid Dynamic Shape Optimization

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Optimal Shape Design (OSD) in the context of fluid flow exposed geometries aims at a shape that minimizes a given objective functional. Using a steepest descent approach, the required sensitivities are preferably obtained by adjoint methods due to the independence of the computational costs from the number of design variables. In the past, usually single-phase flows were addressed. In this case typical challenges of the adjoint approach, such as transient processes or discontinuities, do not necessarily occur. For immiscible two-phase flows, however, these issues can no longer be avoided since the flow is inherently unsteady and afflicted with discontinuities along the (sharp) free surface. The latter is captured with a Volume-of-Fluid (VoF) method, which is of particular relevance due to its wide application in marine engineering simulations.

As observed for a simple Couette flow, the corresponding boundary objective based set of adjoint equations is unfortunately ill-posed. An additional heuristic diffusive adjoint concentration term, which was previously introduced into the adjoint VoF system, might serve as a remedy to this issue. Although this term regularizes the solution of the adjoint equation system, it violates the dual consistency [1]. The talk will focus on an alternative immiscible two-phase formulation. The Cahn-Hilliard equation describes the process of phase separation of different fluids. It can be employed for the to describe the evolution of the mixture fraction for primal two-phase flow equations. In general, the approach can be understood as an extension of the standard primal VoF approach with a nonlinear right hand side of order four. Based on two additional model parameters, a scalable, continuous and thus differentiable phase transition regime arises. This finally yields a consistent and well posed adjoint system.

Results obtained by a numerical implementation of the CH-RANS approach will be *verified* against analytical primal as well as adjoint solutions, belonging to a force objective. Subsequently, a generic benchmark case is studied and the resulting sensitivities are *validated* against results obtained from finite differencing. The talk closes with the presentation of three-dimensional simulations for the (primal/adjoint) flow around a ship hull at Reynolds and Froude numbers of practical interest.

## REFERENCES

- [1] Kröger, J., Kühl, N., Rung, T. Adjoint Volume-of-Fluid Approaches for the Hydrodynamic Optimisation of Ships. *Ship Technology Research* (2018) 65, pp. 47 – 68