

REVISION OF BEAM MODELING WITH LARGE ROTATIONS

Anthoula N. Panteli¹ and Konstantinos V. Spiliopoulos²

¹ Doctoral Student, School of Civil Engineering, National Technical University of Athens, Zografou Campus, 157-80 Athens, Greece, anpanteli@central.ntua.gr

² Professor, School of Civil Engineering, National Technical University of Athens, Zografou Campus, 157-80 Athens, Greece, kvspilio@central.ntua.gr

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Beam modeling is used in the deformation and motion analysis of slender structural members. In this work, the state of the art concerning the essential rod theory in non-linear regime is summed up. The basic assumption is that the plane of the cross-section remains planar and rigid, and the material is homogeneous, isotropic and linear elastic.

A geometrical illustration on the rotation manifold $SO(3)$ is given, through the comparison of two formulations describing the orientation change of the cross-section, the first that uses additive rotation increments [1] and the second that uses spin increments [2]. This geometrical presentation clarifies the construction of an invariant formulation with large rotations. The invariance properties, i.e. the objectivity and path-independence, are satisfied when spins are used as the unknown parameters. In this case, the whole rotation is shown on the manifold as it is split into an elemental and a local cross-sectional spin relatively to the element.

Two finite element formulations, based on the previous two concepts of parametrization of rotations, are derived and used in benchmark examples. In the second one, the interpolation is performed for the local spins of the nodes, which refer to the same tangent space; this idea leads to the invariance of the formulation. The test cases show clearly that the strain energy is not distorted, neither due to the rigid motion (objectivity), nor due to the path that has been followed (path-independence).

Thus, the comparison between the two formulations, will help a researcher, who is ensuing research in this area, to realize the numerical procedures that have to be followed in the construction of any invariant model with large rotations.

REFERENCES

- [1] E. N. Dvorkin, E. Oñate, J. Oliver, On a non-linear formulation for curved Timoshenko beam elements considering large displacement/rotation increments, *International Journal for Numerical Methods in Engineering*, Vol. **26**, pp. 1597-1613, 1988.
- [2] G. Jelenić, M. A. Crisfield, Geometrically exact 3D beam theory: implementation of a strain-invariant finite element for statics and dynamics, *Computer Methods in Applied Mechanics and Engineering*, Vol. **171**, pp. 141-171, 1999.