

# A high-order scheme for weakly compressible flows

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The high-order approximation of compressible fluid flow in the low Mach number regime is cumbersome on many levels. First, the Euler (and Navier-Stokes) equations at low Mach number  $M$  constitute a singularly perturbed system of equations as the speed of sound tends to infinity. This renders the issue of stability of an overall numerical method quite delicate. Second, even if the method is stable, the phenomenon of order reduction can occur, meaning that the desired formal order of accuracy is only obtained when highly resolving the flow, but not in the intermediate regime where an efficient computation should be performed. Third, of course all the other intricacies that make high-order methods particularly difficult remain.

In this talk, we present a discontinuous Galerkin solver with implicit/explicit (IMEX) time integration. It is discussed how at least some of the raised issues can be tackled. In particular, we focus on a recently developed splitting relying on the incompressible reference solution [1, 2, 3]. We discuss the issue of stability based on numerical analysis for prototypical equations, see, e.g., [1, 4], and based on numerical results for more realistic problems. Furthermore, we discuss the issue of very high-order IMEX schemes.

## REFERENCES

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